THE EFFECT OF ERROR NON-NORMALITY ON THE POWER OF PARAMETRIC AND NON-PARAMETRIC ANOV TESTS

ROBERT WILLIAM GERMANY JONES

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by

Robert William Germany Jones

Thesis Advisor: T. D. Burnett

September 1971

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on the Power of Parametric and Non-parametric ANOV Tests

by

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ABSTRACT

The purpose of this thesis is to determine the power relationship, through computer simulation, between the parametric ANOV and non-parametric Wilson tests under controlled conditions of error non-normality.

Data is simulated using the 12 cell factorial ANOV model with three levels of factor A, four levels of factor B, and six observations per cell. Interaction is characterized such that its effect is proportional to the effect of factor A with the constant of proportionality related to factor B. Non-normality of the error term is characterized in three distribution types: skewed, leptokurtic (peaked), and platykurtic (flat). Four degrees of the three error distribution types are utilized, each related to the Pearson family of frequency curves.

Three thousand-seven hundred sets of data are generated for each degree of error type. Power is then estimated directly for both the ANOV F tests and Wilson Chi-square tests for main effects and interaction. Comparison is then made between corresponding tests showing the effect of error non-normality on the power of each.

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I. INTRODUCTION

A. PEARSON'S STUDY OF NON-NORMAL VARIATION

The normality of error assumption is a well known requirement when using Analysis of Variance (ANOV) techniques. It has been shown, however, that this requirement is not stringent when only type I error is the point of concern. The literature does suggest that error normality is a requirement relative to type II error and correct determination of power of the test.

Pearson [1] studied error non-normality for a case involving the one way ANOV classification. His study was based on the distribution of the correlation ratio η^2 which, as used, was equivalent to the F statistic. Six error distributions were chosen with non-normality of error characterized by Pearson coefficients β_1 and β_2[1] and by Pearson curve type[2]. Those chosen were:

$\beta_1 = 0.0$ $\beta_2 = 2.50$ (Type II, symmetrical platykurtic),

$\beta_1 = 0.0$ $\beta_2 = 4.10$ (Type VII, symmetrical leptokurtic),

$\beta_1 = 0.0$ $\beta_2 = 7.05$ (Type VII, symmetrical leptokurtic),

$\beta_1 = 0.2$ $\beta_2 = 3.3$ (Type III, skew),

$\beta_1 = 0.49$ $\beta_2 = 3.72$ (Type III, skew),

$\beta_1 = 0.99$ $\beta_2 = 3.83$ (Type I, very skew with abrupt start).

Pearson concluded that the distribution of η^2, and therefore

---

[1] Reference 2 defines these coefficients as $\beta_1 = \mu_3^3/\mu_2^3$, $\beta_2 = \mu^4/\mu_2^2$, where μ_i represents the ith central moment.

[2] Reference 3 transforms the 13 curve types of Reference 2 into probability density functions.

of F, within the range of the above six distributions adequately met the "normality of error" requirement for Analysis of Variance. He further concluded that within this range there would be little chance of rejecting a true null hypothesis because of non-normality, but that in the extreme cases of non-normal variation there would always be a danger of accepting a false null hypothesis. Thus, Pearson suggested that extreme cases of error non-normality may result in a reduction of power in the F test. He thereby raised the question as to how error non-normality would affect the F test power for a particular ANOV design.

Kirk [4], citing the above study by Pearson (1931) and a study by Norton as reported by Lindquist [5], extended Pearson's conclusion relating to type I error to all fixed-effects ANOV models utilizing the F distribution. Kirk held that, in general, unless the departure from normality is so extreme that it can be readily detected by visual inspection of the data, the departure will have little effect on the probability associated with the test of significance (type I error). However, Kirk made no reference to the effect of non-normality on the power of the test.

B. A NON-PARAMETRIC ANOV TEST

The problem of meeting the error normality assumption for using the parametric ANOV F tests is avoided by choosing a non-parametric test. A search of the literature was made to find such a test which included a non-parametric method for testing interactions in the ANOV model. Wilson [6]

developed such a test, based on the Chi-square distribution, for testing hypothesis in two-way, three-way, up to n-way ANOV designs. The test procedure, applicable only to fixed effects models, involves classifying the scores in each cell as above or below the overall median and using the fact that a total Chi-square, like a sum of squares, can be decomposed into additive parts. A new set of formulas are introduced for working with factorial designs in Chi-square terms. A description of the test including the Chi-square formulas is contained in Appendix A.

Sheffield [7] showed how the Wilson test could be converted to a conventional ANOV procedure, whereby the Wilson Chi-square formulation could be replaced with "non-parametric F" tests.

McNemar [8] contrasted the outcomes of the Wilson test and the parametric F test on seven batches of data, each involving two-way classification. Based on 21 comparisons (A, B, and AxB effects) he suggested that the power of the Wilson tests was lower than what could be reasonably expected. However, McNemar failed to determine what constituted "reasonable" power for the Wilson test. In addition, he based his conclusions on a small sample which he admittedly assumed met the parametric ANOV requirements of normality and homoscadacity. To the author's knowledge, the literature does not report any other attempt to obtain an indication of power for the Wilson Chi-square tests.

C. TYPE OF INTERACTION

Williams [9] studied the problem of interpreting the effects of different factors when those effects are not additive. He pointed out that attention must be paid to the way in which the factors interact. He suggested that a reasonable assumption, in the two factor case, was to consider that the interaction effect was proportional to the effect of one factor with the constant of proportionality related to the second factor (i.e., $(\alpha\beta)_{ij} = \alpha_i c_j$). Williams raised the question as to the effect of the type of interaction on power of the tests for hypotheses concerning main effects as well as interaction in the factorial ANOV design.

D. POWER OF THE TEST

Power of the test is defined as the probability of rejecting a false null hypothesis. Power functions have been developed for parametric tests, since these tests are based on assumed known distributions. Power curves for parametric ANOV tests are contained in the Appendix to Reference 10. One of the arguments for entering these curves is ϕ, a function of the factor non-centrality parameter and error variance. Formulas for obtaining the argument ϕ for ANOV designs involving interaction are contained in Reference 11.

Siegel [12] proposed that the power of non-parametric tests can be expressed by comparison with the most powerful existing parametric test that is used for the same purpose since no power functions exist for the "distribution-free" non-parametric tests. Siegel pointed out that the more

general the non-parametric test (the fewer the assumptions) the less powerful the test will be in comparison with a parametric test involving the same sample size. He further stated that the F test, because of its strong assumptions, is the most powerful test of its type. Siegel called his power comparison concept "power efficiency." It is a function of the increase in sample size of the non-parametric test over that of the parametric test which is necessary to make the two tests equally powerful.

Theoretically, the power of a statistical test, parametric or non-parametric, can also be estimated empirically through computer simulation. A simulation model for a particular ANOV design can be constructed and data generated so that the desired factor and interaction effects are present in the data. The null hypothesis, that the "built-in" effect is not present, is then tested at a desired level of significance. Power at the desired level of significance is measured by taking the ratio of the number of times the false null hypothesis is rejected over the number of times the test is conducted.

E. PURPOSE OF THESIS

The purpose of this thesis is to develop a computer simulation model for obtaining a power comparison between the parametric ANOV F tests and the non-parametric Wilson Chi-square tests under varying conditions of unimodal error non-normality. It is envisaged that the general method will be applicable in determining the power of other non-parametric tests.

II. METHOD

A. ANOV DESIGN AND GENERAL PLAN

A 3x4 ANOV design with six replications per cell was chosen because of its general nature and because it was desired to compare interaction detection capabilities of the Wilson and F tests, as well as that of main effects. The method developed is presented in five parts: characterizing error, characterizing interaction, the data simulation model, the computer program, and determination of test replications for desired confidence.

B. CHARACTERIZING ERROR

Only unimodal error was considered. Error was characterized as skewed, leptokurtic, and platykurtic. Four degrees of each type were considered, from violent to almost normal. The degenerate case for each error distribution type was the normal error distribution and this was considered as a fifth degree. Each degree was identified by Pearson coefficients β_1 and β_2. For continuity with Pearson's work, some degrees were chosen identical to the error distributions studied in Reference 1. All error distributions were selected with a mean of 0 and a variance of 4.

C. CHARACTERIZING INTERACTION

Interaction was characterized as follows:

$$(\alpha\beta)_{ij} = \alpha_i c_j \qquad \begin{array}{l} i=1,\cdot\cdot,3 \\ j=1,\cdot\cdot,4 \end{array} \qquad (1)$$

where c_j's were constants, evenly spaced, increasing and $\sum_j c_j = 0$. (Note that the "evenly spaced" and "increasing" features are not required.) This characterization resulted in the interaction effect, $(\alpha\beta)_{ij}$, being proportional to the effect of factor A, α_i, and increasing with each level of factor B.

D. THE DATA SIMULATION MODEL

1. The Mathematical Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + z_{ijk} \qquad (2)$$

where

$i = 1,\cdot\cdot,3;\ j = 1,\cdot\cdot,4;\ k = 1,\cdot\cdot,6.$

z_{ijk} represents the error which under the ANOV assumption is distributed $N(0,\sigma^2)$. It was the distribution of this error term that was changed from normality, but with a constant variance of $\sigma^2 = 4$, in order to determine the effect of non-normality on the Wilson and F tests. The development of equation (2) into the twelve cell models shown below is contained in Appendix B.

Model for cell one:

$$Y_{11k} = \mu + \alpha_1 + \beta_1 + (\alpha\beta)_{11} + z_{11k} \qquad k=1,\cdot\cdot,6. \quad (3)$$

Model for cell two:

$$Y_{12k} = \mu + \alpha_1 + \beta_2 + (\alpha\beta)_{12} + z_{12k} \qquad k=1,\cdot\cdot,6. \quad (4)$$

Model for cell three:

$$Y_{13k} = \mu + \alpha_1 + \beta_3 + (\alpha\beta)_{13} + z_{13k} \qquad k=1,\cdot\cdot,6. \quad (5)$$

Model for cell four:

$$Y_{14k} = \mu + \alpha_1 - \beta_1 - \beta_2 - \beta_3 - (\alpha\beta)_{11} - (\alpha\beta)_{12}$$

$$- (\alpha\beta)_{13} + z_{14k} \qquad k=1,\cdot\cdot,6. \quad (6)$$

Model for cell five:

$$Y_{21k} = \mu + \alpha_2 + \beta_1 + (\alpha\beta)_{21} + z_{21k} \qquad k=1,\cdot\cdot,6. \quad (7)$$

Model for cell six:

$$Y_{22k} = \mu + \alpha_2 + \beta_2 + (\alpha\beta)_{22} + z_{22k} \qquad k=1,\cdot\cdot,6. \quad (8)$$

Model for cell seven:

$$Y_{23k} = \mu + \alpha_2 + \beta_3 + (\alpha\beta)_{23} + z_{23k} \qquad k=1,\cdot\cdot,6. \quad (9)$$

Model for cell eight:

$$Y_{24k} = \mu + \alpha_2 - \beta_1 - \beta_2 - \beta_3 - (\alpha\beta)_{21} - (\alpha\beta)_{22}$$

$$- (\alpha\beta)_{23} + z_{24k} \qquad k=1,\cdot\cdot,6. \quad (10)$$

Model for cell nine:

$$Y_{31k} = \mu - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21}$$

$$+ z_{31k} \qquad k=1,\cdot\cdot,6. \quad (11)$$

Model for cell ten:

$$Y_{32k} = \mu - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22}$$

$$+ z_{32k} \qquad k=1,\cdot\cdot,6. \quad (12)$$

Model for cell eleven:

$$Y_{33k} = \mu - \alpha_1 - \alpha_2 + \beta_3 - (\alpha\beta)_{13} - (\alpha\beta)_{23}$$

$$+ z_{33k} \qquad k=1,\cdot\cdot,6. \quad (13)$$

Model for cell twelve:

$$Y_{34k} = \mu - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 - \beta_3 + (\alpha\beta)_{11} + (\alpha\beta)_{12}$$

$$+ (\alpha\beta)_{13} + (\alpha\beta)_{21} + (\alpha\beta)_{22} + (\alpha\beta)_{23} + z_{34k}$$

$$k=1,\cdot\cdot,6. \quad (14)$$

Procedures for generating error variates z_{ijk} and for determining the parameter values $\alpha_1, \alpha_2, \cdots, (\alpha\beta)_{23}$ are given below. It may be noted that several of the parameters of equation (2) have dropped out of the twelve cell models as explained in Appendix B.

2. Generation of Error, z_{ijk}

a. Normal Error Distribution

Normal error variates, with mean of 0 and variance of 4, were generated by using the central limit approach contained in Reference 13. A normal variate z was made by applying the following simulation formula:

$$z = \sigma_z \left(\frac{12}{k}\right)^{\frac{1}{2}} \sum_{i=1}^{k} \left(r_i - \frac{k}{2}\right) + \mu_z \qquad (15)$$

where:

μ_z = desired mean of the normal error variate (here=0),

σ_z = desired standard deviation of the normal error variate (here=2),

r = a Uniform (0,1) random number, and

k = number of random variates desired for approximating a normal variate by the central limit approach. A value of

k=12 was chosen for convenience which reduced (15) to

$$z = \sigma_z \left(\sum_{i=1}^{12} r_i - 6.0 \right). \tag{16}$$

b. Skewed Error Distribution

The most violent degree of skewed error was an exponential distribution and was generated using the exponential generator shown in Reference 13. Advantage was taken of the fact that

$$r = \exp \left(- \frac{1}{\mu_z} z \right),$$

and

$$z = - \mu_z \log r, \tag{17}$$

where z in this case is an exponential variate with desired mean of μ_z and desired variance of $\sigma_z^2 = \mu_z^2 = 4$. The variate z was then transformed by the below formula to obtain a mean of 0.

$$z' = z - \mu_z.$$

The other three degrees of skewed error were generated with the gamma (erlang) generator of Reference 13. The simulation formula was developed from the probability density function (pdf)

$$f(z) = \frac{\alpha^k \; z^{(k-1)} \; e^{\alpha z}}{(k-1)!} \tag{18}$$

where $\alpha > 0$, k is a positive integer, and z is a non-negative erlang variate. Here

$$\alpha = \frac{\mu_z}{\sigma_z^2}$$

and

$$k = \frac{\mu_z^2}{\sigma_z^2} .$$

Erlang variates were made by taking the sum of k exponential variates,

$$z = -\frac{1}{\alpha} \sum_{i=1}^{k} \log r_i ,$$

which is equivalent to

$$z = -\frac{1}{\alpha} \left(\log \prod_{i=1}^{k} r_i \right). \tag{19}$$

Since μ_z had to be greater than 0 in the simulation equation (19), the below transformation was used to transform the variate so that the resulting error distribution had a mean of 0.

$$z' = z - \mu_z . \tag{20}$$

The three combinations of α and k chosen to generate the three levels of erlang variates with mean 0 and variance 4 were:

α	k
2.0	16
1.0	4
$\sqrt{2}/2$	2.

c. Leptokurtic Error Distribution

The four degrees of leptokurtic error were generated by sampling from an empirical cumulative distribution function (cdf). The below pdf equation for a type VII

Pearson curve (leptokurtic) was obtained from Reference 3.

$$y = y_o \left(1 + \frac{x^2}{a^2}\right)^{-m} \tag{21}$$

where

$$m = \frac{5\beta_2 - 9}{2(\beta_2 - 3)} ,$$

$$a^2 = \frac{2\mu_2\beta_2}{\beta_2 - 3} \quad \text{and } \mu_2 = \text{desired variance} = 4,$$

$$y_o = \frac{N}{a\sqrt{\pi}} \cdot \frac{\Gamma(m)}{\Gamma(m-\frac{1}{2})} \quad \text{and } N = \text{number of desired variates in the distribution}$$

and

x = abscissa index value for the associated ordinate density value, y.

One hundred index values from -12.0 to + 12.0, six standard deviations, were used in making an empirical pdf by computer simulation. A routine was incorporated into the program that calculated the mean, variance, unbiased estimates $\hat{\beta}_1$ and $\hat{\beta}_2$, and frequencies at the 100 index points (i.e., histogram frequencies). This was done to insure that the error generated was accurate. The pdf was converted to a cdf by successively summing the pdf values along the same horizontal axis (-12.0 to +12.0). A Uniform (0,1) random number was then generated and used to sample from the empirical cdf. A binary search technique was incorporated in the computer program at this "sampling" stage to reduce computer time. The error variate was determined by interpolating between the two index points on the horizontal axis which bracketed the

probability value from the cdf that corresponded to the sampling random number.

d. Platykurtic Error Distribution

The four degrees of platykurtic error were generated by a similar empirical cdf sampling method. The pdf equation determined from Reference 3 for a type II curve was:

$$y = y_o \left(1 - \frac{x^2}{a^2}\right)^m \tag{22}$$

where

$$m = \frac{5\beta_2 - 9}{2(3-\beta_2)} ,$$

$$a^2 = \frac{2\mu_2 \beta_2}{3 - \beta_2} , \text{ and}$$

$$y_o = \frac{N}{a\sqrt{\pi}} \cdot \frac{\Gamma(m+1\tfrac{1}{2})}{\Gamma(m+1)} .$$

A similar routine for calculating mean, variance, $\hat{\beta}_1$, $\hat{\beta}_2$, and histogram frequencies was also included in the computer program for platykurtic error (and also for skewed and normal error) to insure that the results were as desired. Selected computer programs are attached following the appendices to this thesis. Smooth histogram curves for the degrees of each error type are shown in Appendix C.

3. Model Parameter Values

The Wilson test was the first claimed distribution free method for testing interactions in the ANOV model. Therefore, determining the power of such an interaction test was a primary consideration. It was desired to chose parameters for the 3x4 model (equations (3) through (14)) which would

guarantee that the power of the parametric F test for interaction would be in a sensitive range on the ANOV power curve. After studying the appropriate power curve, shown in Appendix D, a desired interaction test power of .67 was chosen. This gave a ϕ (curve argument) value of 1.3. Working backwards from this, subject to the conditions given in Appendix B and the chosen interaction characterization, the below parameter values were determined for equations (3) through (14). See Appendix D for development and computations.

$$\mu = 100.00$$
$$\alpha_1 = -1.03$$
$$\alpha_2 = 0.28$$
$$\beta_1 = -1.03$$
$$\beta_2 = -1.03$$
$$\beta_3 = 1.03$$
$$(\alpha\beta)_{11} = 1.4832$$
$$(\alpha\beta)_{12} = 0.4944$$
$$(\alpha\beta)_{13} = 0.4944$$
$$(\alpha\beta)_{21} = -0.4032$$
$$(\alpha\beta)_{22} = -0.1344$$
$$(\alpha\beta)_{23} = 0.1344.$$

E. THE COMPUTER PROGRAM

The generation of error variates was incorporated into the main computer program. A different "package" was written for each degree of each error distribution type. Six different

replication of error variates were then applied in each of the 12 cell models to produce 72 observations for testing by the parametric ANOV F tests and the Wilson Chi-square tests.

Separate subroutines were written for the parametric ANOV and Wilson tests. F and Chi-square statistic values were computed for main and interaction effects.

In the main program these computed values were compared to the threshold values from the F and Chi-square tables at the 5 per cent level of significance. If the calculated F or Chi-square value was greater than or equal to the threshold value, the false null hypothesis was rejected and "counted." The total count was then divided by the number of times the test was conducted to obtain the power of the test.

Flow charts of the main program, subroutine ANOV, and subroutine Wilson are shown in Appendix E.

F. DETERMINATION OF TEST REPLICATIONS FOR DESIRED CONFIDENCE

Stein (1945) showed how a sequential sampling procedure could be used for establishing a confidence interval of fixed length e for estimating the mean μ having a confidence coefficient $\geq 1-\alpha$.

This procedure was used to establish the number of test replications (N) necessary to obtain at least 95 per cent confidence of being within ±0.02 ($e = 0.04$) in determining power of the test for the three F tests and three Chi-square tests.

Stein's theorem and proof are contained in Reference 14. Application of the theorem to obtain N is shown in Appendix F. The computer program utilized is attached following the appendices. The result was N = 3700. This was the maximum of the replications required for the six tests. The deciding test was the Wilson Chi-square test for the A (row) effect.

III. VALIDATION

A. DETERMINATION OF THEORETICAL POWER, ANOV TESTS

As stated above, a desired power of 0.67 for the ANOV interaction test was used in determining the simulation model parameters. Using the below equations from Reference 11,

$$\phi_A^2 = \frac{nc \sum_{i=1}^{r} \alpha_i^2}{\sigma^2 r} \quad \text{and} \qquad (23)$$

$$\phi_B^2 = \frac{nr \sum_{j=1}^{c} \beta_j^2}{\sigma^2 c} , \qquad (24)$$

and substituting applicable parameter values, theoretical powers for the testing the A and B main effects by ANOV F tests were found to be 0.81 and 0.96 respectively. ϕ calculations and the applicable ANOV power curves are contained in Appendix G.

B. ANOV POWER SIMULATION RESULTS, N(0,4) ERROR

Three thousand-seven hundred data cases with N(0,4) error and the predetermined model parameters were tested with the ANOV subroutine. Power was computed and compared with theoretical power.

EFFECT	A	B	AxB
Theoretical W/ N(0,4) Error	0.81	0.96	0.67
Simulated W/ N(0,4) Error	0.808	0.960	0.665

C. ACCURACY CHECK OF WILSON SUBROUTINE

One hundred data cases with N(0,4) error and predetermined model parameters were tested with the WILSON subroutine. Printout was made on median calculation, contingency table, and Chi-square values (total, A, B, AxB). Five of the cases were then selected at random and the same values were calculated by hand. Results were exactly the same.

D. SIMULATION OF LEVELS OF SIGNIFICANCE

Parameters of the simulation model, other than the overall mean, were set equal to zero. Then 3700 data sets were produced with N(0,4) error and tests conducted with the ANOV and WILSON subroutines. Based on this sample size the true level of significance was estimated within plus or minus 0.02 with 95 per cent confidence. That is, by choosing F and Chi-square threshold values for rejection at the 5 and 10 per cent significance level, the chance percentage of rejections should have been equivalent, within the criteria, to 0.05 and 0.10 for each of the ANOV F and Wilson Chi-square tests. Results were:

5 PER CENT REJECTION THRESHOLD

EFFECTS	A	B	AxB
ANOV F	.0565	.0535	.0508
WILSON χ^2	.0586	.0486	.0411

10 PER CENT REJECTION THRESHOLD

EFFECTS	A	B	AxB
ANOV F	.1051	.1089	.0995
WILSON χ^2	.0754	.0884	.0954

IV. RESULTS

Three thousand-seven hundred data cases for each of the four degrees of skewed, leptokurtic, and platykurtic error were tested with the ANOV and WILSON subroutines. Results are given below in three tables. Each table gives the power of the test for the three effects hypotheses with a particular type error. Pearson coefficients are shown for reference, as well as the actual means and variances of the error distributions generated. The last column of each table gives the power of the test with N(0,4) error for convenience of comparison. Error degrees shown correspond to the curves of Appendix C.

	ERROR DEGREES				
	1	2	3	4	5
SKEWED	$\beta_1=3.97$	$\beta_1=1.95$	$\beta_1=0.99$	$\beta_1=0.26$	$\beta_1=0.00$
ERROR	$\beta_2=9.03$	$\beta_2=5.96$	$\beta_2=4.57$	$\beta_2=3.47$	$\beta_2=2.95$
DIST'N	(EXPON)				(NORMAL)
COMPUTED DIST'N MEAN	0.00	0.00	0.00	0.00	0.00
COMPUTED DIST'N VARIANCE	3.97	3.98	3.98	3.97	3.98
$H_o: \ \Sigma \alpha_i^2 = 0$					
$H_A: \ \Sigma \alpha_i^2 \neq 0$					
POWER ANOV	0.82	0.81	0.82	0.82	0.81
POWER WILSON	0.74	0.64	0.58	0.51	0.47
$H_o: \ \Sigma \beta_j^2 = 0$					
$H_A: \ \Sigma \beta_j^2 \neq 0$					
POWER ANOV	0.95	0.95	0.96	0.96	0.96
POWER WILSON	0.97	0.91	0.85	0.78	0.74
$H_o: \ \Sigma\Sigma (\alpha\beta)_{ij} = 0$					
$H_A: \ \Sigma\Sigma (\alpha\beta)_{ij} \neq 0$					
POWER ANOV	0.68	0.67	0.66	0.66	0.66
POWER WILSON	0.36	0.30	0.27	0.25	0.25

	ERROR DEGREES				
	1	2	3	4	5
LEPTOKURTIC	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$
ERROR	$\beta_2=20.00$	$\beta_2=7.05$	$\beta_2=4.10$	$\beta_2=3.40$	$\beta_2=2.95$
DIST'N					(NORMAL)
COMPUTED DIST'N MEAN	-0.10	-0.11	-0.12	-0.12	0.00
COMPUTED DIST'N VARIANCE	3.86	3.93	3.96	3.97	3.98
$H_o: \Sigma \alpha_i^2 = 0$					
$H_A: \Sigma \alpha_i^2 \neq 0$					
POWER ANOV	0.82	0.81	0.81	0.81	0.81
POWER WILSON	0.56	0.53	0.50	0.48	0.47
$H_o: \Sigma \beta_j^2 = 0$					
$H_A: \Sigma \beta_j^2 \neq 0$					
POWER ANOV	0.96	0.95	0.96	0.96	0.96
POWER WILSON	0.86	0.82	0.78	0.75	0.74
$H_o: \Sigma\Sigma (\alpha\beta)_{ij} = 0$					
$H_A: \Sigma\Sigma (\alpha\beta)_{ij} \neq 0$					
POWER ANOV	0.68	0.67	0.66	0.66	0.66
POWER WILSON	0.30	0.29	0.26	0.25	0.25

	ERROR DEGREES				
	1	2	3	4	5
PLATYKURTIC	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$	$\beta_1=0.00$
ERROR	$\beta_2=1.80$	$\beta_2=2.00$	$\beta_2=2.25$	$\beta_2=2.50$	$\beta_2=2.95$
DIST'N	(UNIFORM)				(NORMAL)
COMPUTED DIST'N MEAN	0.00	-0.03	-0.05	-0.06	0.00
COMPUTED DIST'N VARIANCE	3.97	3.97	3.97	3.97	3.98
$H_o: \Sigma \alpha_i^2 = 0$					
$H_A: \Sigma \alpha_i^2 \neq 0$					
POWER ANOV	0.81	0.81	0.82	0.82	0.81
POWER WILSON	0.37	0.40	0.42	0.44	0.47
$H_o: \Sigma \beta_j^2 = 0$					
$H_A: \Sigma \beta_j^2 \neq 0$					
POWER ANOV	0.96	0.96	0.96	0.96	0.96
POWER WILSON	0.55	0.60	0.65	0.68	0.74
$H_o: \Sigma\Sigma (\alpha\beta)_{ij} = 0$					
$H_A: \Sigma\Sigma (\alpha\beta)_{ij} \neq 0$					
POWER ANOV	0.65	0.65	0.65	0.65	0.66
POWER WILSON	0.18	0.20	0.21	0.23	0.25

V. ANALYSIS AND INTERPRETATION OF RESULTS

A. HYPOTHESIS TEST POWER CURVES

Figures 1, 2, and 3, which are attached following this section, compare the power of the ANOV F and Wilson Chi-square tests by the null hypothesis being tested. Each figure contains three sets of curves, one for each error distribution type. The horizontal argument of "error degree" corresponds to the degree of non-normal error distribution type shown in the distribution curves of Appendix C.

B. ANOV F TESTS

Figures 1 through 3 show that for a 3x4 factorial with interaction and 6 observations per cell the power of the F test for main effects and interaction effects hypotheses are unaffected by the three types of error non-normality. The range of power change from the estimated power with normal error is only - 0.01 to + 0.02 over all three hypotheses. It appears that there should be no concern relative to degradation of the power of the F test because of error non-normality, even in extreme cases, as long as the data is unimodal.

C. WILSON CHI-SQUARE TESTS

The below curves indicate that the Wilson Chi-square tests are not "distribution free" as claimed. The power of the tests for main effects and interaction effects are sensitive to the shape of the error distribution.

The normal error power estimate for the Wilson A effect test was 0.47. Non-normal error in that the test resulted

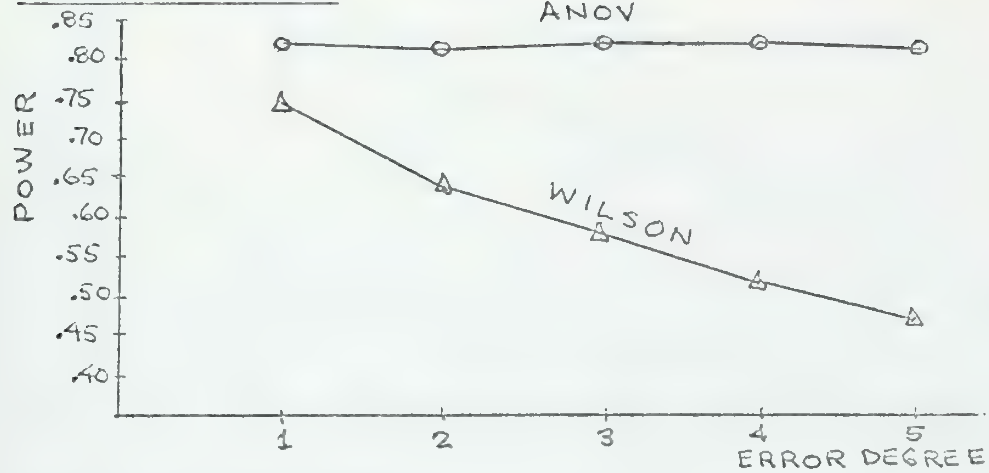
in a range of power change of -0.10 (violent platykurtic) to +0.27 (violent skewed) from the normal value. For the B effect test the range of power change from the normal error estimate of 0.74 was -0.19 (violent platykurtic) to +0.23 (violent skewed). For the AxB effect test the range of power change from a norm of 0.25 was -0.07 to +0.11, with the same error type extremes as before.

In general as expected the power of the Wilson Chi-square tests were lower than the power of the comparable F tests. When the error distribution was N(0,4) for both, the power of the Chi-square test for A effect was 58 per cent of the power of the comparable F test, the power of the Chi-square test for B effect was 77 per cent of the power of the F test, and the power of the Chi-square test for AxB effect was 38 per cent of that of the F test. However, there was one case where the power of the Chi-square test was higher than that of the F test (0.97 versus 0.95 in the test for B effect under the condition of violent skewed error).

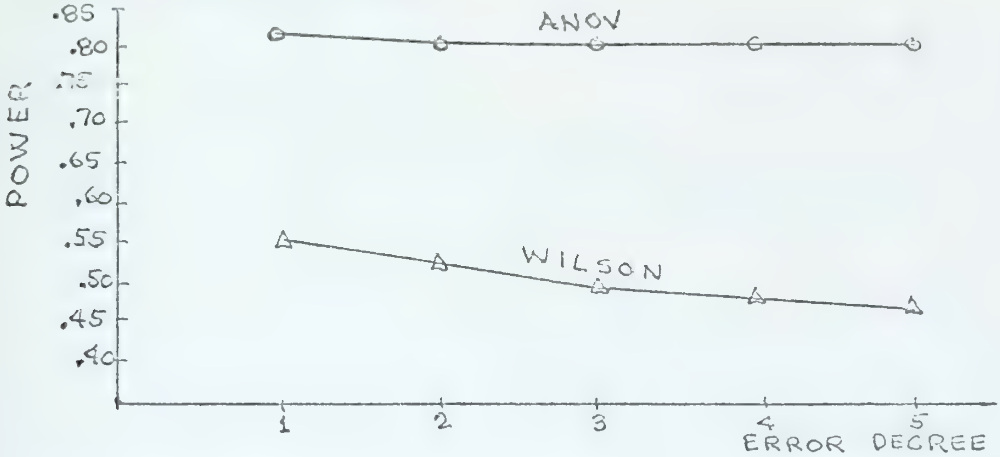
The error effect trend for a particular error type was consistent over the three hypotheses tested. Note the similarity in shape, for example, of the skewed error Wilson power curve in Figures 1 through 3. The rate of change in power with respect to error degree (as error varied from normality) was, however, significantly less for the Chi-square test for interaction under the three error types than it was in the Chi-square tests for both main effects under equivalent error types.

FIGURE 1. $\underline{H_0: \sum_i \alpha_i^2 = 0}$

SKEWED ERROR



LEPTOKURTIC ERROR



PLATYKURTIC ERROR

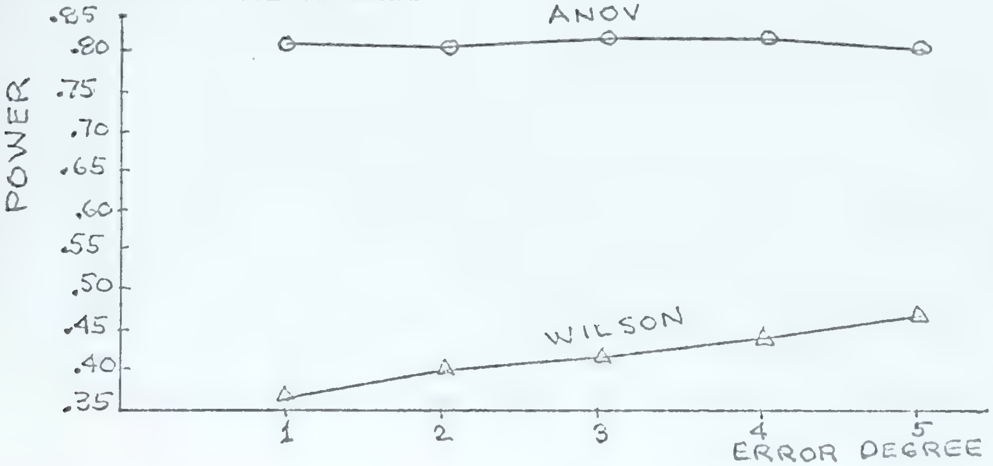
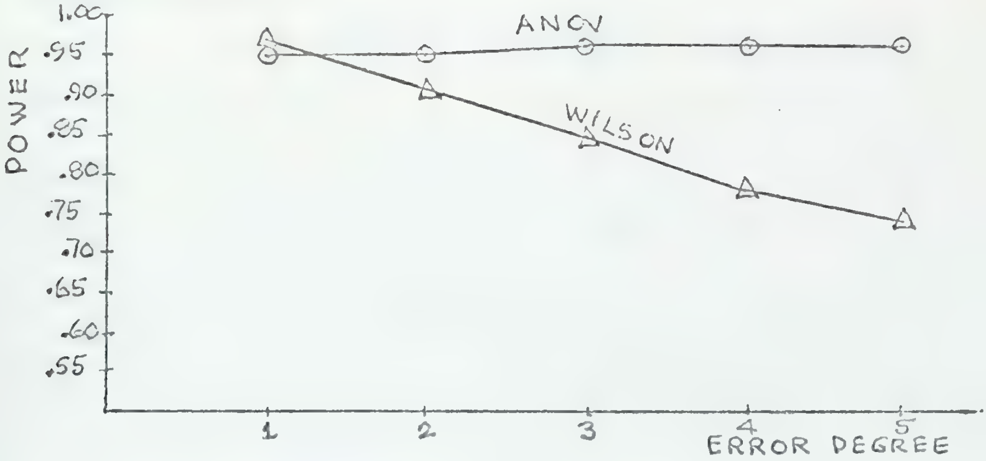
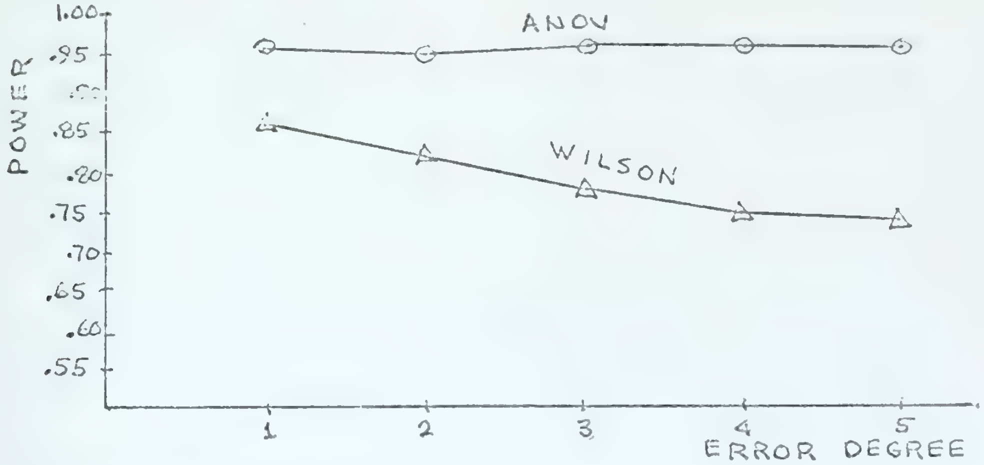


FIGURE 2. $H_0: \sum_j \beta_j^2 = 0$


SKEWED ERROR


LEPTOKURTIC ERROR

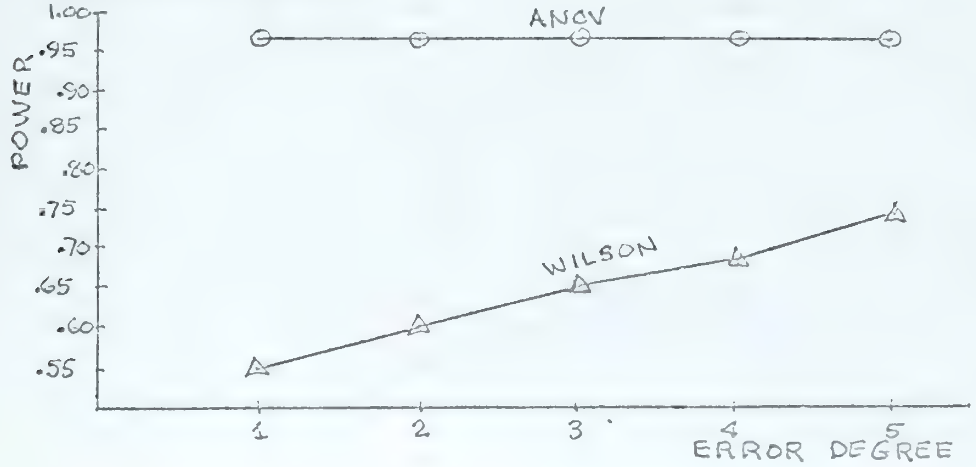
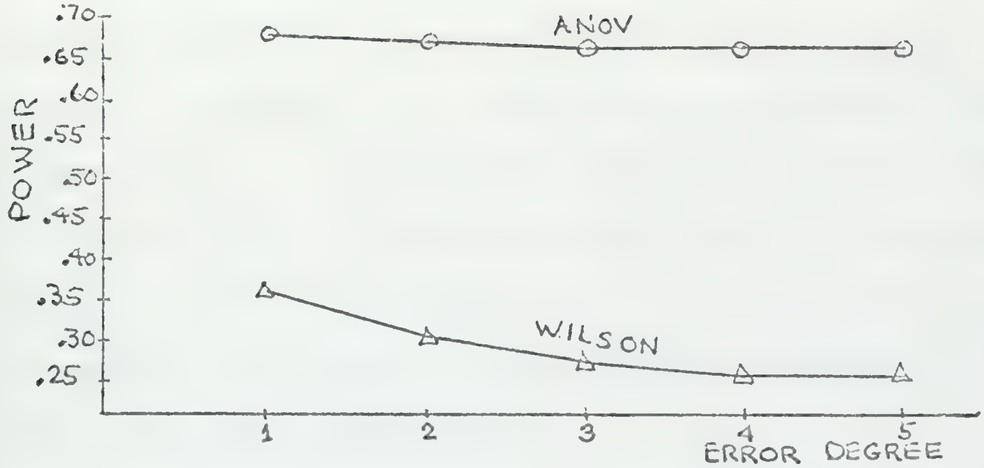

PLATYKURTIC ERROR

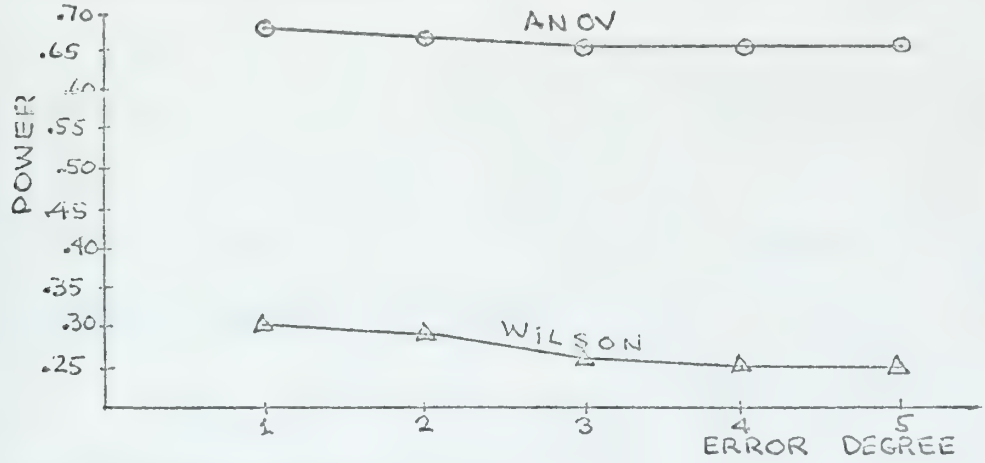
FIGURE 3.

$H_0: \sum_i \sum_j (\alpha\beta)_{ij} = 0$

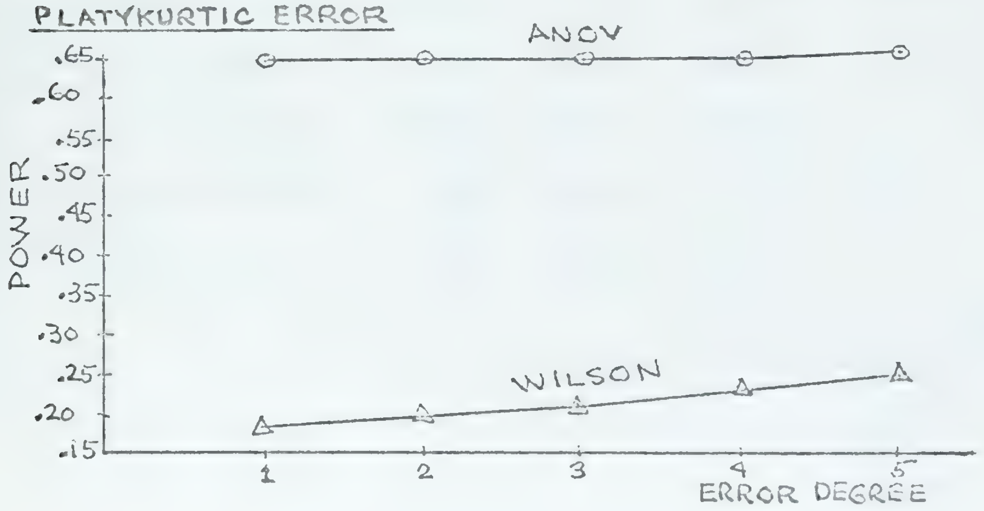
SKEWED ERROR



LEPTOKURTIC ERROR



PLATYKURTIC ERROR



VI. DISCUSSION

A. WILSON TEST SHAPE SENSITIVITY

It is apparent from Figures 1 through 3 that the distributions of the Wilson Chi-square test statistics under the alternate hypotheses are sensitive to the shape of the error distribution. The question arises as to whether the distributions of these statistics under the null hypotheses are also sensitive to the shape of the error distribution; that is, whether the type I error (alpha) is changing with each degree of error distribution.

To investigate this, level of significance estimates were made under the four degrees of skewed error distribution by determining power with the model coefficients set equal to zero and Chi-square threshold values set for alpha = 0.05. Results were obtained and compared to level of significance results with N(0,4) error.

	A	B	AxB
WILSON SKEW #1	.0624	.0524	.0489
WILSON SKEW #2	.0659	.0530	.0432
WILSON SKEW #3	.0659	.0530	.0432
WILSON SKEW #4	.0649	.0524	.0454
WILSON N(0,4)	.0586	.0486	.0411

Although the true levels of significance were slightly higher under the skewed error, they were higher by approximately the same amount in all cases. On this basis it is concluded that

the distributions of the Wilson Chi-square statics are only sensitive to the error distribution under the alternate hypotheses.

Since the test as proposed by Wilson uses the Chi-square as the approximate distribution of the test statistics it is evident that this approximation does hold under the null hypothesis (at least in the right hand tail of the distribution) but that the degree of approximation of the non-central Chi-square under the alternate hypothesis is sensitive to changes in the error distribution. Sawrey [15] has labeled such tests as "semi-nonparametric."

B. USE OF THE WILSON TEST

The above should not be interpreted to mean that the Wilson test is invalid and should be avoided. It is obviously inferior to the ANOV F test and should not be chosen when the data permits utilization of the parametric ANOV. However, the Wilson test, to the author's knowledge, is the simplest of only two procedures applicable to testing for interaction when the data is qualitative and measurements have been obtained only on an ordinal scale (see Mood [16] for the other). It should be used in such cases with the realization that its accuracy depends upon the shape of the underlying error distribution. After histogramming the data, the results of this paper may be used to provide general insight relative to the question of what power to expect. Pertinent power estimates for a particular design may be obtained by employing the computer simulation method presented herein,

provided that the error distribution (unimodal) can be identified from the data and the type interaction can be characterized. Twenty-eight and one half minutes on an IBM 360 computer were required on the average in this study for obtaining power estimates for the normal case and one non-normal error distribution.

C. AREAS FOR FURTHER STUDY

1. Power Efficiency

A typical question facing a prospective user of the Wilson test is that of how many replications are necessary with a particular design to attain a desired power, given a particular error distribution based on preliminary sampling. This is a problem in estimating power efficiency for the Wilson test and is a natural extension of this study. The same simulation model could be used.

2. Different Type of Interaction

The present research could be extended to include other characterizations of interaction. In the course of this study the author at one stage characterized interaction by $(\alpha\beta)_{ij} = \alpha_i c_{ij} + b_j$. Incomplete results, based on only 200 batches of data, indicated that the estimated power for the F tests was the same as that for the characterization stated herein (i.e., $(\alpha\beta)_{ij} = \alpha_i c_j$), but that the estimated power for the Wilson Chi-square tests was considerably different from that under the present characterization.

3. Effect of Homoscadacity on Power

It is believed that a computer simulation method, similar to the one used here relative to non-normality, could

be used to study the effect on power of the parametric ANOV requirement of homoscadacity. Error distributions could be generated with different degrees of error variance, data tested by the ANOV and WILSON subroutines, and the results analyzed.

4. Semi-Nonparametric Tests

Another research area recommended is that of "semi-nonparametric" tests. An appropriate thesis might consist of extracting all such tests from the literature and then showing why they are semi-nonparametric with resulting implications. Reference 15 would be a good starting point for such a thesis.

5. Effect of Sample Size on Non-normality Effect

A final extension of this study might be to analyze the effect of sample size (replications per cell) on the power of the ANOV F and Wilson Chi-square tests under the same conditions of error non-normality. Would a doubling or tripling of sample size tend to flatten out the Wilson power curves shown in Figures 1, 2, and 3 of Section V or would they just be shifted upwards? Would the ANOV F tests still be unaffected by error non-normality if the sample size were only two replications per cell?

VII. SUMMARY

A complete simulation method has been presented for estimating power for both parametric ANOV tests and Wilson non-parametric ANOV tests. A 3x4 ANOV simulation model was used with six replications per cell. Interaction was characterized by $(\alpha\beta)_{ij} = \alpha_i c_j$ with c_j's constant, increasing, evenly spaced, and $\sum_j c_j = 0$. Error non-normality was characterized in four degrees of three unimodal error distribution types: skewed, leptokurtic, and platykurtic. It was shown that when the error was normal the Wilson Chi-square test for A effect was 58 per cent of that for the comparable F test, the Chi-square test for B effect was 77 per cent of that for the F test, and the Chi-square test for AxB effect was 38 per cent of that for the comparable F test. It was further shown that the Wilson Chi-square tests were not distribution free as claimed by Wilson but were sensitive to the parent distribution shape. Leptokurtic and skewed error distributions increased the power of the Chi-square tests above that estimated with normal error. Platykurtic error distributions decreased the power of the Chi-square tests from that estimated with normal error. The power of the ANOV F tests were unaffected by even the extreme cases of error non-normality.

APPENDIX A: DESCRIPTION OF THE WILSON TEST

1. The median value, M_d, for the entire set of n observations is determined. The number of observations less than M_d, represented by n_b, is then calculated.

2. A 2xrxc contingency table is constructed where r and c represent the number of rows and columns of the design and the "third dimension 2" corresponds to the division of scores by M_d. The frequency entries for the contingency table are represented by ${}_b f_{ij}$, the number of observations less than M_d for the cell in row i and column j of the table. It follows that

$$n_b = \sum_i \sum_j {}_b f_{ij}. \qquad (1-1)$$

The below contingency table example is given for the purpose of clarifying notation:

$M_d = 100.54851$

		FACTOR A				
		1	2	3	4	${}_b f_{i\cdot}$
FACTOR B	1	5	6	2	2	15
	2	3	2	4	2	11
	3	4	4	1	1	10
	${}_b f_{\cdot j}$	12	12	7	5	36

all

$n_{ij} = 6$

$r = 3$

$c = 4$

${}_b f_{ij}$ = # observations $< M_d$

$n_b = 36$

$n = 72.$

3. Since the number of observations for each cell in the ANOV design of interest are all equal and M_d can be calculated such that $n_b = n/2$, the total Chi-square value can be computed as follows:

$$\chi^2_T = \left(\frac{4rc}{n}\right) \sum_i \sum_j \left({}_b f_{ij} - \frac{n}{2rc}\right)^2 \tag{1-2}$$

where n/2rc represents the expected frequency under the null hypothesis that the main effects and interaction effects produce no change in the distribution of scores. χ^2_T has (rc-1) degrees of freedom.

4. The Chi-square values of the row effects and column effects are computed using the marginal totals of the 2xrxc contingency table.

$$\chi^2_R = \left(\frac{4r}{n}\right) \sum_i \left({}_b f_{i\cdot} - \frac{n}{2r}\right)^2 \tag{1-3}$$

and

$$\chi^2_C = \left(\frac{4c}{n}\right) \sum_j \left({}_b f_{\cdot j} - \frac{n}{2c}\right)^2 \tag{1-4}$$

where ${}_b f_{i\cdot} = \sum_i {}_b f_{ij}$ and ${}_b f_{\cdot j} = \sum_j {}_b f_{ij}$. As before, the expected frequencies for the main effects, n/2r and n/2c, are obtained for the null hypotheses that the distributions of scores are identical for all levels of the row or column

effects. χ_R^2 and χ_C^2 have (r-1) and (c-1) degrees of freedom respectively.

5. The Chi-square value for the interaction effect is computed by subtraction.

$$\chi_I^2 = \chi_T^2 - \chi_R^2 - \chi_C^2 \qquad (1\text{-}5)$$

χ_I^2 has (r-1)(c-1) degrees of freedom.

6. The tests for the main effects and interaction are made by comparing the obtained values of χ_R^2, χ_C^2, and χ_I^2 with values from the cumulative Chi-square distribution for the appropriate degrees of freedom and desired significance level.

7. Wilson [6], citing Rao (1952) and Cochran (1954) concluded that there was no problem concerning small expected frequencies as long as the contingency table has 30 or more degrees of freedom. He further concluded that ordinary Chi-square tables were applicable as long as the 30 degrees of freedom criteria held.

8. Wilson (6) included formulation for Chi-square values when n_a, the number of observations greater than or equal to M_d, is not equal n_b and when the n_{ij} are not all equal. In addition he extended application of his test to experimental designs with other than two factors.

APPENDIX B: DEVELOPMENT OF 12 CELL MODELS

1. The Mathematical Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + z_{ijk}$$

$$i=1,..,3;\ j=1,..,4;\ k=1,..,6$$

is expressed in matrix notation as

$$Y \quad = \quad X \quad B \quad + \quad Z$$

$$\begin{pmatrix} Y_{111} \\ \vdots \\ Y_{116} \\ Y_{121} \\ \vdots \\ Y_{126} \\ Y_{131} \\ \vdots \\ Y_{136} \\ Y_{141} \\ \vdots \\ Y_{146} \\ Y_{211} \\ \vdots \\ Y_{216} \\ Y_{221} \\ \vdots \\ Y_{226} \\ Y_{231} \\ \vdots \\ Y_{236} \\ Y_{241} \\ \vdots \\ Y_{246} \\ Y_{311} \\ \vdots \\ Y_{316} \\ Y_{321} \\ \vdots \\ Y_{326} \\ Y_{331} \\ \vdots \\ Y_{336} \\ Y_{341} \\ \vdots \\ Y_{346} \end{pmatrix} = \left(\begin{array}{cccccccccccccccccccc}
1&1&0&0&1&&&&1 \\
\vdots&\vdots&\vdots&\vdots&\vdots&&&&\vdots \\
1&1&0&0&1&&&&1 \\
1&1&0&0&&1&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&\vdots&&&&\vdots \\
1&1&0&0&&1&&&&1 \\
1&1&0&0&&&1&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&\vdots&&&&\vdots \\
1&1&0&0&&&1&&&&1 \\
1&1&0&0&&&&1&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&&\vdots&&&&\vdots \\
1&1&0&0&&&&1&&&&1 \\
1&0&1&0&1&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&\vdots&&&&&&&&\vdots \\
1&0&1&0&1&&&&&&&&1 \\
1&0&1&0&&1&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&\vdots&&&&&&&&\vdots \\
1&0&1&0&&1&&&&&&&&1 \\
1&0&1&0&&&1&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&\vdots&&&&&&&&\vdots \\
1&0&1&0&&&1&&&&&&&&1 \\
1&0&1&0&&&&1&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&&\vdots&&&&&&&&\vdots \\
1&0&1&0&&&&1&&&&&&&&1 \\
1&0&0&1&1&&&&&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&\vdots&&&&&&&&&&&&\vdots \\
1&0&0&1&1&&&&&&&&&&&&1 \\
1&0&0&1&&1&&&&&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&\vdots&&&&&&&&&&&&\vdots \\
1&0&0&1&&1&&&&&&&&&&&&1 \\
1&0&0&1&&&1&&&&&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&\vdots&&&&&&&&&&&&\vdots \\
1&0&0&1&&&1&&&&&&&&&&&&1 \\
1&0&0&1&&&&1&&&&&&&&&&&&1 \\
\vdots&\vdots&\vdots&\vdots&&&&\vdots&&&&&&&&&&&&\vdots \\
1&0&0&1&&&&1&&&&&&&&&&&&1
\end{array}\right) \begin{pmatrix} \mu \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \alpha\beta_{11} \\ \alpha\beta_{12} \\ \alpha\beta_{13} \\ \alpha\beta_{14} \\ \alpha\beta_{21} \\ \alpha\beta_{22} \\ \alpha\beta_{23} \\ \alpha\beta_{24} \\ \alpha\beta_{31} \\ \alpha\beta_{32} \\ \alpha\beta_{33} \\ \alpha\beta_{34} \end{pmatrix} + \begin{pmatrix} Z_{111} \\ \vdots \\ Z_{116} \\ Z_{121} \\ \vdots \\ Z_{126} \\ Z_{131} \\ \vdots \\ Z_{136} \\ Z_{141} \\ \vdots \\ Z_{146} \\ Z_{211} \\ \vdots \\ Z_{216} \\ Z_{221} \\ \vdots \\ Z_{226} \\ Z_{231} \\ \vdots \\ Z_{236} \\ Z_{241} \\ \vdots \\ Z_{246} \\ Z_{311} \\ \vdots \\ Z_{316} \\ Z_{321} \\ \vdots \\ Z_{326} \\ Z_{331} \\ \vdots \\ Z_{336} \\ Z_{341} \\ \vdots \\ Z_{346} \end{pmatrix}$$

(72x1) (72x20) (20x1) (72x1)

2. Model conditions are:

$$\alpha_1 + \alpha_2 + \alpha_3 = 0 \qquad (2\text{-}2)$$

$$\beta_1 + \beta_2 + \beta_3 = 0 \qquad (2\text{-}3)$$

$$(\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{13} + (\alpha\beta)_{14} = 0 \qquad (2\text{-}4)$$

$$(\alpha\beta)_{21} + (\alpha\beta)_{22} + (\alpha\beta)_{23} + (\alpha\beta)_{24} = 0 \qquad (2\text{-}5)$$

$$(\alpha\beta)_{31} + (\alpha\beta)_{32} + (\alpha\beta)_{33} + (\alpha\beta)_{34} = 0 \qquad (2\text{-}6)$$

$$(\alpha\beta)_{11} + (\alpha\beta)_{21} + (\alpha\beta)_{31} = 0 \qquad (2\text{-}7)$$

$$(\alpha\beta)_{12} + (\alpha\beta)_{22} + (\alpha\beta)_{32} = 0 \qquad (2\text{-}8)$$

$$(\alpha\beta)_{13} + (\alpha\beta)_{23} + (\alpha\beta)_{33} = 0 \qquad (2\text{-}9)$$

$$(\alpha\beta)_{14} + (\alpha\beta)_{24} + (\alpha\beta)_{34} = 0 \qquad (2\text{-}10)$$

3. Rewriting the above and eliminating duplication:

$$\alpha_3 = -\alpha_1 - \alpha_2 \qquad (2\text{-}11)$$

$$\beta_4 = -\beta_1 - \beta_2 - \beta_3 \qquad (2\text{-}12)$$

$$(\alpha\beta)_{14} = -(\alpha\beta)_{11} - (\alpha\beta)_{12} - (\alpha\beta)_{13} \qquad (2\text{-}13)$$

$$(\alpha\beta)_{24} = -(\alpha\beta)_{21} - (\alpha\beta)_{22} - (\alpha\beta)_{23} \qquad (2\text{-}14)$$

$$(\alpha\beta)_{31} = -(\alpha\beta)_{11} - (\alpha\beta)_{21} \qquad (2\text{-}15)$$

$$(\alpha\beta)_{32} = -(\alpha\beta)_{12} - (\alpha\beta)_{22} \qquad (2\text{-}16)$$

$$(\alpha\beta)_{33} = -(\alpha\beta)_{13} - (\alpha\beta)_{23} \qquad (2\text{-}17)$$

$$(\alpha\beta)_{34} = -(\alpha\beta)_{14} - (\alpha\beta)_{24} \qquad (2\text{-}18)$$

Thus the matrix equation can be reduced to an equation in only 12 parameters (vice 20).

4. The new new matrix equation is expressed on page 43. The following cell models are obtained by expanding the new matrix equation.

Model for cell one:

$$Y_{11k} = \mu + \alpha_1 + \beta_1 + (\alpha\beta)_{11} + z_{11k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}19)$$

Model for cell two:

$$Y_{12k} = \mu + \alpha_1 + \beta_2 + (\alpha\beta)_{12} + z_{12k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}20)$$

Model for cell three:

$$Y_{13k} = \mu + \alpha_1 + \beta_3 + (\alpha\beta)_{13} + z_{13k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}21)$$

Model for cell four:

$$Y_{14k} = \mu + \alpha_1 - \beta_1 - \beta_2 - \beta_3 - (\alpha\beta)_{11}$$

$$- (\alpha\beta)_{12} - (\alpha\beta)_{13} + z_{14k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}22)$$

Model for cell five:

$$Y_{21k} = \mu + \alpha_2 + \beta_1 + (\alpha\beta)_{21} + z_{21k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}23)$$

Model for cell six:

$$Y_{22k} = \mu + \alpha_2 + \beta_2 + (\alpha\beta)_{22} + z_{22k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}24)$$

Model for cell seven:

$$Y_{23k} = \mu + \alpha_2 + \beta_3 + (\alpha\beta)_{23} + z_{23k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}25)$$

Model for cell eight:

$$Y_{24k} = \mu + \alpha_2 - \beta_1 - \beta_2 - \beta_3 - (\alpha\beta)_{21}$$

$$- (\alpha\beta)_{22} - (\alpha\beta)_{23} + z_{24k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}26)$$

Model for cell nine:

$$Y_{31k} = \mu - \alpha_1 - \alpha_2 + \beta_1 - (\alpha\beta)_{11} - (\alpha\beta)_{21}$$

$$+ z_{31k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}27)$$

$$
\underset{(72x1)}{\left[\begin{array}{c}
Y_{111}\\ \vdots\\ Y_{116}\\ \hline
Y_{121}\\ \vdots\\ Y_{126}\\ \hline
Y_{131}\\ \vdots\\ Y_{136}\\ \hline
Y_{141}\\ \vdots\\ Y_{146}\\ \hline
Y_{211}\\ \vdots\\ Y_{216}\\ \hline
Y_{221}\\ \vdots\\ Y_{226}\\ \hline
Y_{231}\\ \vdots\\ Y_{236}\\ \hline
Y_{241}\\ \vdots\\ Y_{246}\\ \hline
Y_{311}\\ \vdots\\ Y_{316}\\ \hline
Y_{321}\\ \vdots\\ Y_{326}\\ \hline
Y_{331}\\ \vdots\\ Y_{336}\\ \hline
Y_{341}\\ \vdots\\ Y_{346}
\end{array}\right]}
=
\underset{(72x12)}{\left(\begin{array}{cccccccccccc}
1&1&0&1&&&1&&&&&\\
\vdots&\vdots&\vdots&\vdots&&&\vdots&&&&&\\
1&1&0&1&&&1&&&&&\\
1&1&0&&1&&&1&&&&\\
\vdots&\vdots&\vdots&&\vdots&&&\vdots&&&&\\
1&1&0&&1&&&1&&&&\\
1&1&0&&&1&&&1&&&\\
\vdots&\vdots&\vdots&&&\vdots&&&\vdots&&&\\
1&1&0&&&1&&&1&&&\\
1&1&0&-1&-1&-1&-1&-1&-1&&&\\
\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&&\\
1&1&0&-1&-1&-1&-1&-1&-1&&&\\
1&0&1&1&&&&&&1&&\\
\vdots&\vdots&\vdots&\vdots&&&&&&\vdots&&\\
1&0&1&1&&&&&&1&&\\
1&0&1&&1&&&&&&1&\\
\vdots&\vdots&\vdots&&\vdots&&&&&&\vdots&\\
1&0&1&&1&&&&&&1&\\
1&0&1&&&1&&&&&&1\\
\vdots&\vdots&\vdots&&&\vdots&&&&&&\vdots\\
1&0&1&&&1&&&&&&1\\
1&0&1&-1&-1&-1&&&&-1&-1&-1\\
\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&&&&\vdots&\vdots&\vdots\\
1&0&1&-1&-1&-1&&&&-1&-1&-1\\
1&-1&-1&1&&&-1&&&-1&&\\
\vdots&\vdots&\vdots&\vdots&&&\vdots&&&\vdots&&\\
1&-1&-1&1&&&-1&&&-1&&\\
1&-1&-1&&1&&&-1&&&-1&\\
\vdots&\vdots&\vdots&&\vdots&&&\vdots&&&\vdots&\\
1&-1&-1&&1&&&-1&&&-1&\\
1&-1&-1&&&1&&&-1&&&-1\\
\vdots&\vdots&\vdots&&&\vdots&&&\vdots&&&\vdots\\
1&-1&-1&&&1&&&-1&&&-1\\
1&-1&-1&-1&-1&-1&1&1&1&1&1&1\\
\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\
1&-1&-1&-1&-1&-1&1&1&1&1&1&1
\end{array}\right)}
\underset{(12x1)}{\left(\begin{array}{c}
\mu\\ \alpha_1\\ \alpha_2\\ \beta_1\\ \beta_2\\ \beta_3\\ \alpha\beta_{11}\\ \alpha\beta_{12}\\ \alpha\beta_{13}\\ \alpha\beta_{21}\\ \alpha\beta_{22}\\ \alpha\beta_{23}
\end{array}\right)}
+
\underset{(72x1)}{\left[\begin{array}{c}
Z_{111}\\ \vdots\\ Z_{116}\\ \hline
Z_{121}\\ \vdots\\ Z_{126}\\ \hline
Z_{131}\\ \vdots\\ Z_{136}\\ \hline
Z_{141}\\ \vdots\\ Z_{146}\\ \hline
Z_{211}\\ \vdots\\ Z_{216}\\ \hline
Z_{221}\\ \vdots\\ Z_{226}\\ \hline
Z_{231}\\ \vdots\\ Z_{236}\\ \hline
Z_{241}\\ \vdots\\ Z_{246}\\ \hline
Z_{311}\\ \vdots\\ Z_{316}\\ \hline
Z_{321}\\ \vdots\\ Z_{326}\\ \hline
Z_{331}\\ \vdots\\ Z_{336}\\ \hline
Z_{341}\\ \vdots\\ Z_{346}
\end{array}\right]}
$$

Model for cell ten:

$$Y_{32k} = \mu - \alpha_1 - \alpha_2 + \beta_2 - (\alpha\beta)_{12} - (\alpha\beta)_{22}$$
$$+ z_{32k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}28)$$

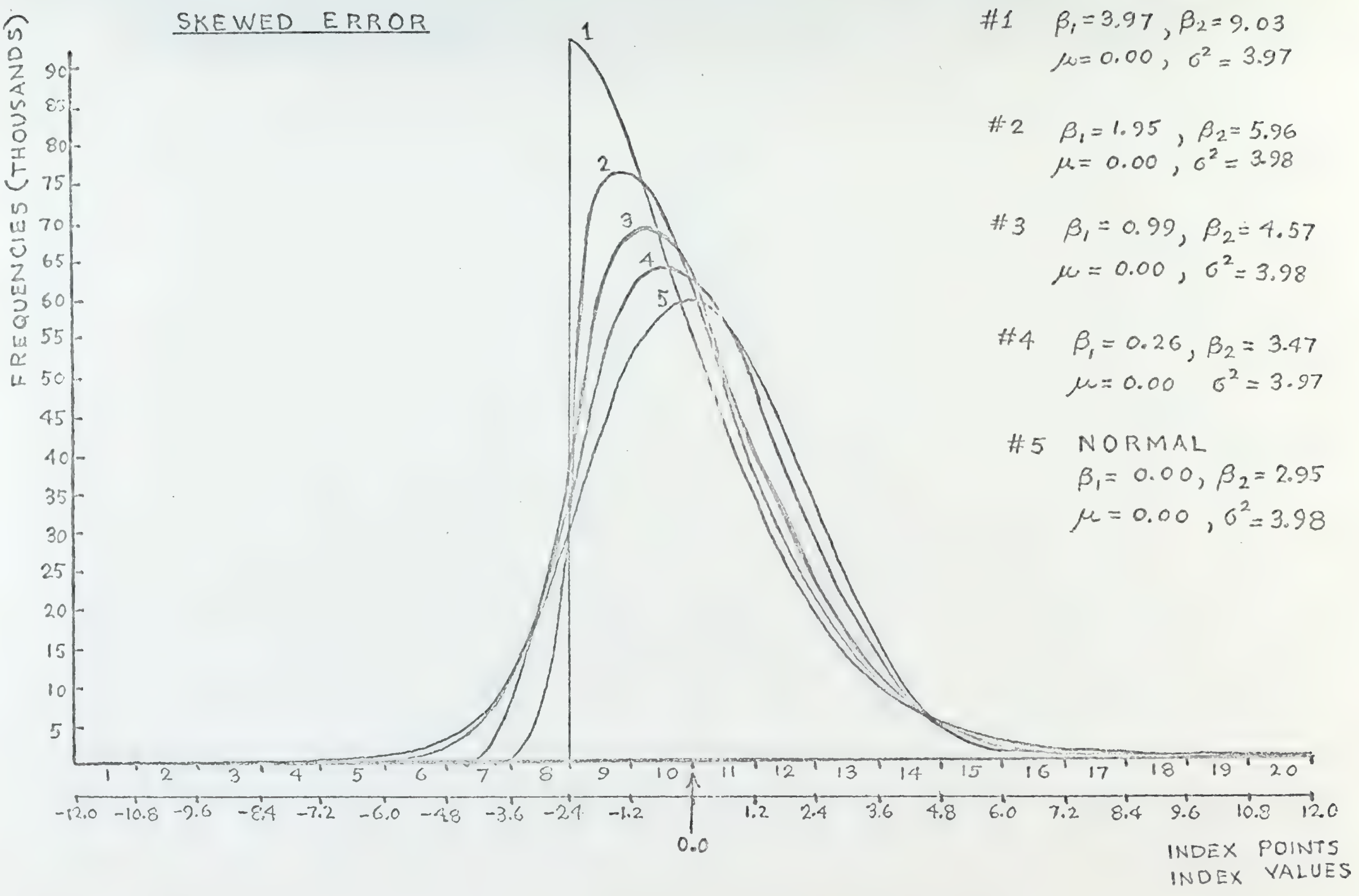
Model for cell eleven:

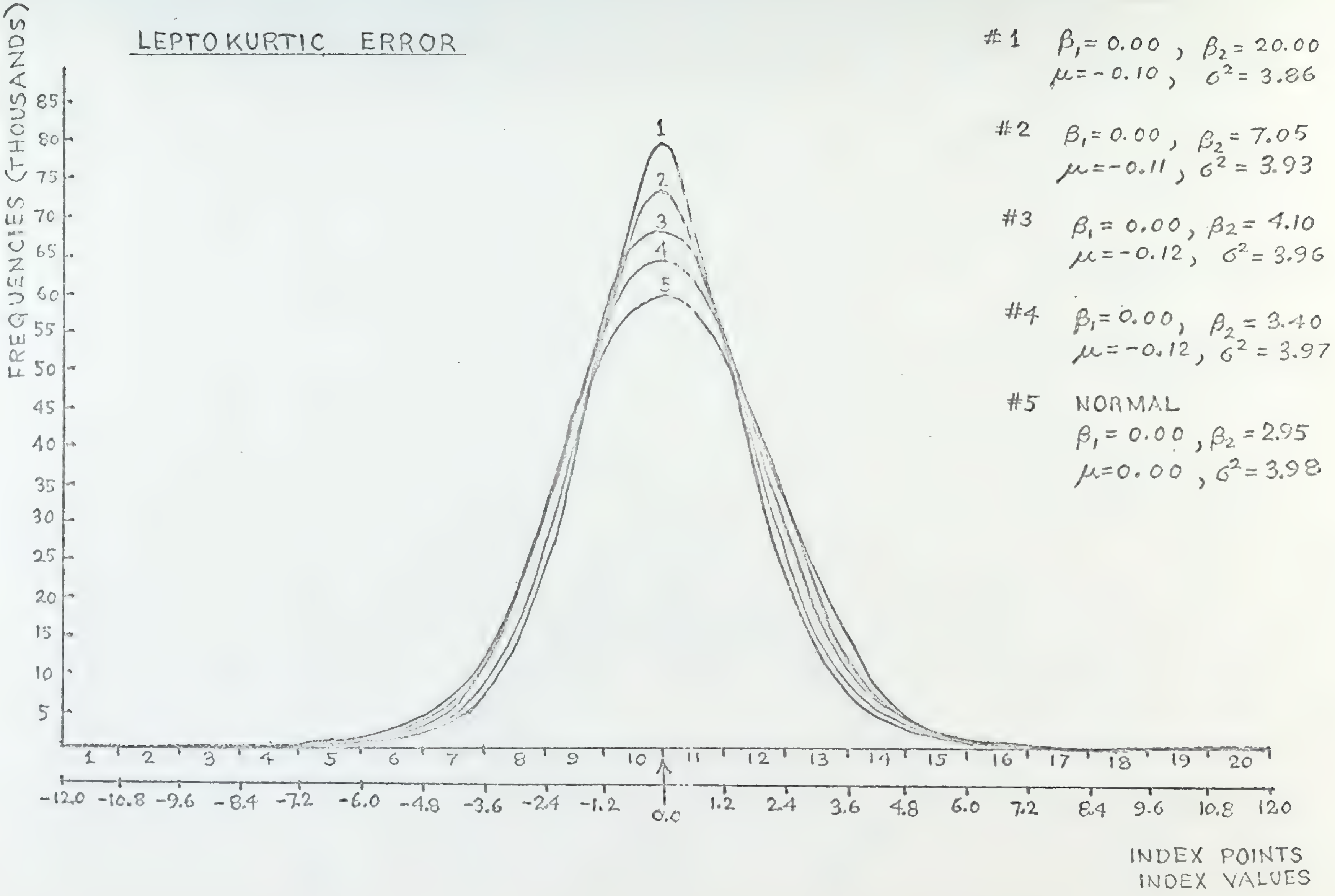
$$Y_{33k} = \mu - \alpha_1 - \alpha_2 + \beta_3 - (\alpha\beta)_{13} - (\alpha\beta)_{23}$$
$$+ z_{33k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}29)$$

Model for cell twelve:

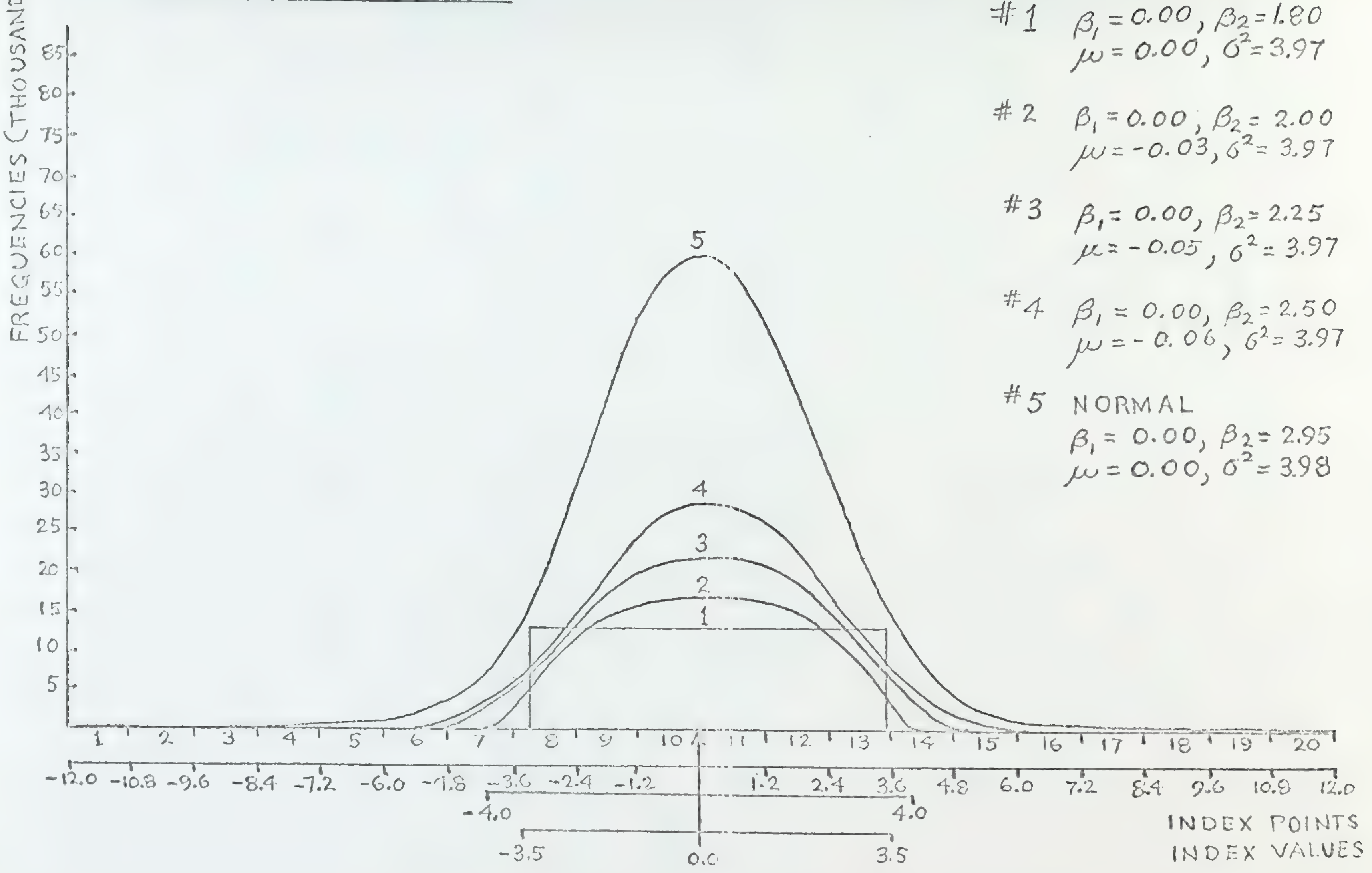
$$Y_{34k} = \mu - \alpha_1 - \alpha_2 - \beta_1 - \beta_2 - \beta_3 + (\alpha\beta)_{11}$$
$$+ (\alpha\beta)_{12} + (\alpha\beta)_{13} + (\alpha\beta)_{21} + (\alpha\beta)_{22}$$
$$+ (\alpha\beta)_{23} + z_{34k} \qquad k=1,\cdot\cdot,6. \quad (2\text{-}30)$$

APPENDIX C: ERROR CURVES




LEPTOKURTIC ERROR
FREQUENCIES (THOUSANDS)
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
1
2
3
4
5
#1 $\beta_1 = 0.00$, $\beta_2 = 20.00$
$\mu = -0.10$, $\sigma^2 = 3.86$
#2 $\beta_1 = 0.00$, $\beta_2 = 7.05$
$\mu = -0.11$, $\sigma^2 = 3.93$
#3 $\beta_1 = 0.00$, $\beta_2 = 4.10$
$\mu = -0.12$, $\sigma^2 = 3.96$
#4 $\beta_1 = 0.00$, $\beta_2 = 3.40$
$\mu = -0.12$, $\sigma^2 = 3.97$
#5 NORMAL
$\beta_1 = 0.00$, $\beta_2 = 2.95$
$\mu = 0.00$, $\sigma^2 = 3.98$
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
-12.0 -10.8 -9.6 -8.4 -7.2 -6.0 -4.8 -3.6 -2.4 -1.2 0.0 1.2 2.4 3.6 4.8 6.0 7.2 8.4 9.6 10.8 12.0
INDEX POINTS
INDEX VALUES

PLATYKURTIC ERROR



APPENDIX D: DEVELOPMENT OF MODEL PARAMETERS

1. The power curve for determining theoretical power for the test of interaction, 3x4 ANOV model, is attached to this appendix. Numerator degrees of freedom are 6, $(r-1)(c-1)$, and denominator degrees of freedom are 60, $rc(n-1)$; where r is the number of rows (3), c is the number of columns (4), and n is the number of replications per cell (6). For a desired power of 0.67 at a level of significance of 0.05, $\phi = 1.3$.

2. From Reference 11

$$\sigma^2_{AxB} = \frac{1}{\sigma^2}\left[\frac{n}{(r-1)(c-1)+1} \sum_{i=1}^{r} \sum_{j=1}^{c} (\alpha\beta)^2_{ij}\right] = (1.3)^2$$

$$= 1.69. \qquad (4\text{-}1)$$

Since a variance of 3.98 was actually obtained in simulation of the N(0,4) error, this value was substituted for σ^2. Values for r, c, and n stated above were also substituted. Then, rearranging (4-1)

$$\sum_{i=1}^{3} \sum_{j=1}^{4} (\alpha\beta)^2_{ij} = \frac{7(3.98)(1.69)}{6} = 7.8387. \qquad (4\text{-}2)$$

3. Any combination of $(\alpha\beta)_{ij}$'s such that (4-2) holds will give the desired power. But the interaction was characterized by $(\alpha\beta)_{ij} = a_i c_j$ with c_j's evenly spaced, constant, increasing with β_j, and $\sum_j c_j = 0$. Initial values for c_j were also arbitrarily chosen as $c_1 = -3$, $c_2 = -1$, $c_3 = 1$, and $c_4 =$

3. Initial values for α_i were also arbitrarily chosen, subject to the condition $\sum_i \alpha_i = 0$, to be $\alpha_1 = -1.03$, $\alpha_2 = 0.28$, $\alpha_3 = 0.75$.

4. Substituting these values for α_i and c_j

$$\sum_i \sum_j (\alpha\beta)_{ij} = \sum_i \sum_j (\alpha_i c_j) = ((-1.03)(-3))$$

$$+\cdots+ ((0.75)(3) = 34.0360.$$

Thus each value of the initial selection for c_j was too large by the multiple $(34.0360/7.8387)^{\frac{1}{2}} = 2.0837$. Initial c_j's were then divided by 2.0837 to obtain:

$$c_1 = -1.43971 = -1.44$$

$$c_2 = -0.47990 = -0.48$$

$$c_3 = 0.47990 = 0.48$$

$$c_4 = 1.43971 = 1.44.$$

5. Using the original α_i's and the adjusted c_j's, values of $(\alpha\beta)_{ij} = (\alpha_i c_j)$ were calculated to be

$$(\alpha\beta)_{11} = \alpha_1 c_1 = 1.4832$$

$$(\alpha\beta)_{12} = \alpha_1 c_2 = 0.4944$$

$$(\alpha\beta)_{13} = \alpha_1 c_3 = -0.4944$$

$$(\alpha\beta)_{14} = \alpha_1 c_4 = -1.4832$$

$$(\alpha\beta)_{21} = \alpha_2 c_1 = -0.4032$$

$$(\alpha\beta)_{22} = \alpha_2 c_2 = -0.1344$$

$$(\alpha\beta)_{23} = \alpha_2 c_3 = 0.1344$$

$$(\alpha\beta)_{24} = \alpha_2 c_4 = 0.4032$$

$$(\alpha\beta)_{31} = \alpha_3 c_1 = -1.0800$$

$$(\alpha\beta)_{32} = \alpha_3 c_2 = -0.3600$$

$$(\alpha\beta)_{33} = \alpha_3 c_3 = 0.3600$$

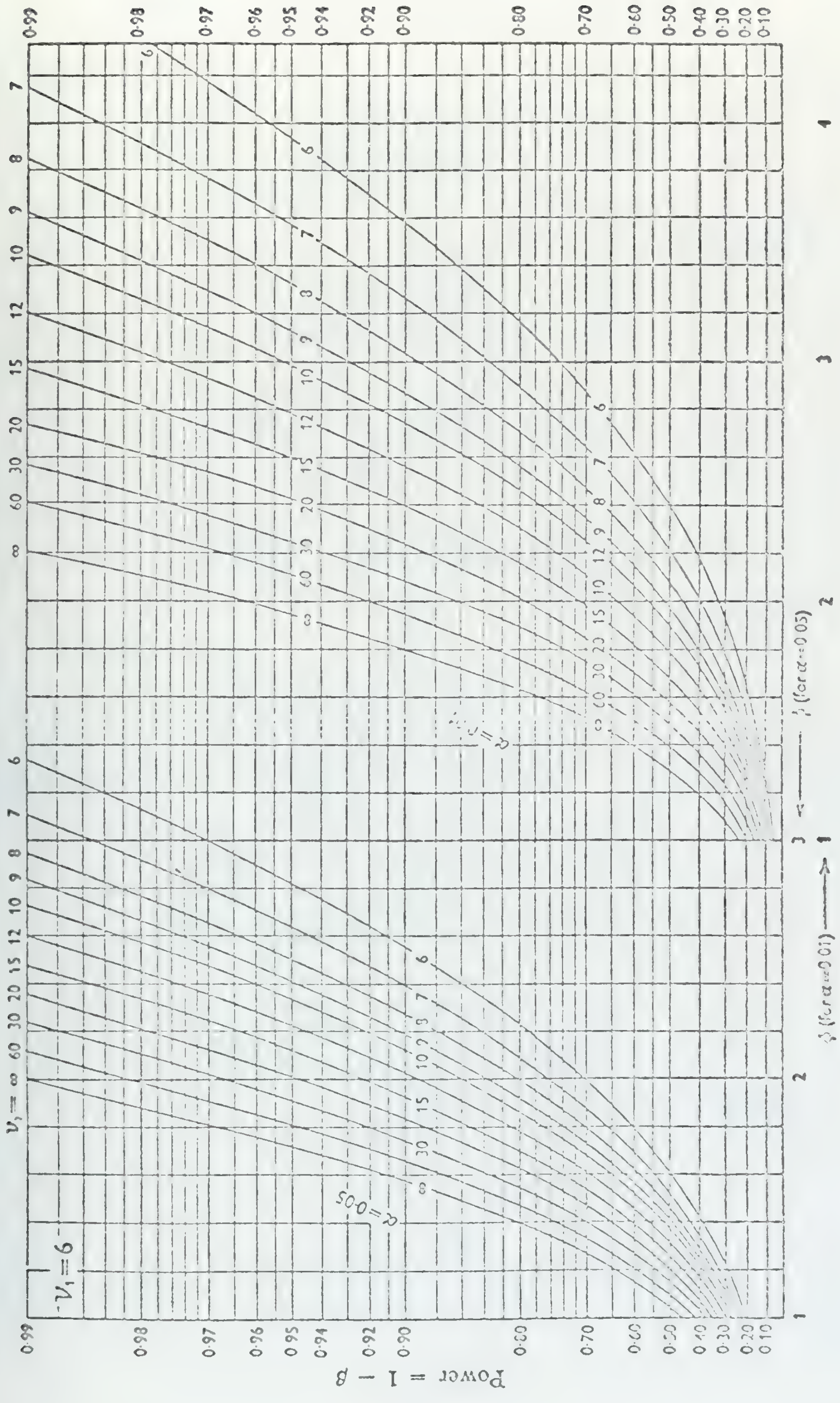
$$(\alpha\beta)_{34} = \alpha_3 c_4 = 1.0800.$$

These values of $(\alpha\beta)_{ij}$ met the model conditions stated in Appendix B (i.e., $\sum_j (\alpha\beta)_{ij} = 0$, $i=1,\cdot\cdot,3$, and $\sum_i (\alpha\beta)_{ij} = 0$, $j=1,\cdot\cdot,4$).

6. Values of β_j were then chosen arbitrarily subject to the condition $\sum_j \beta_j = 0$. Values were selected in the neighborhood of the α_i values so that no single main effect would be dominant. Values chosen were $\beta_1 = -1.03$, $\beta_2 = -1.03$, $\beta_3 = 1.03$, $\beta_4 = 1.03$.

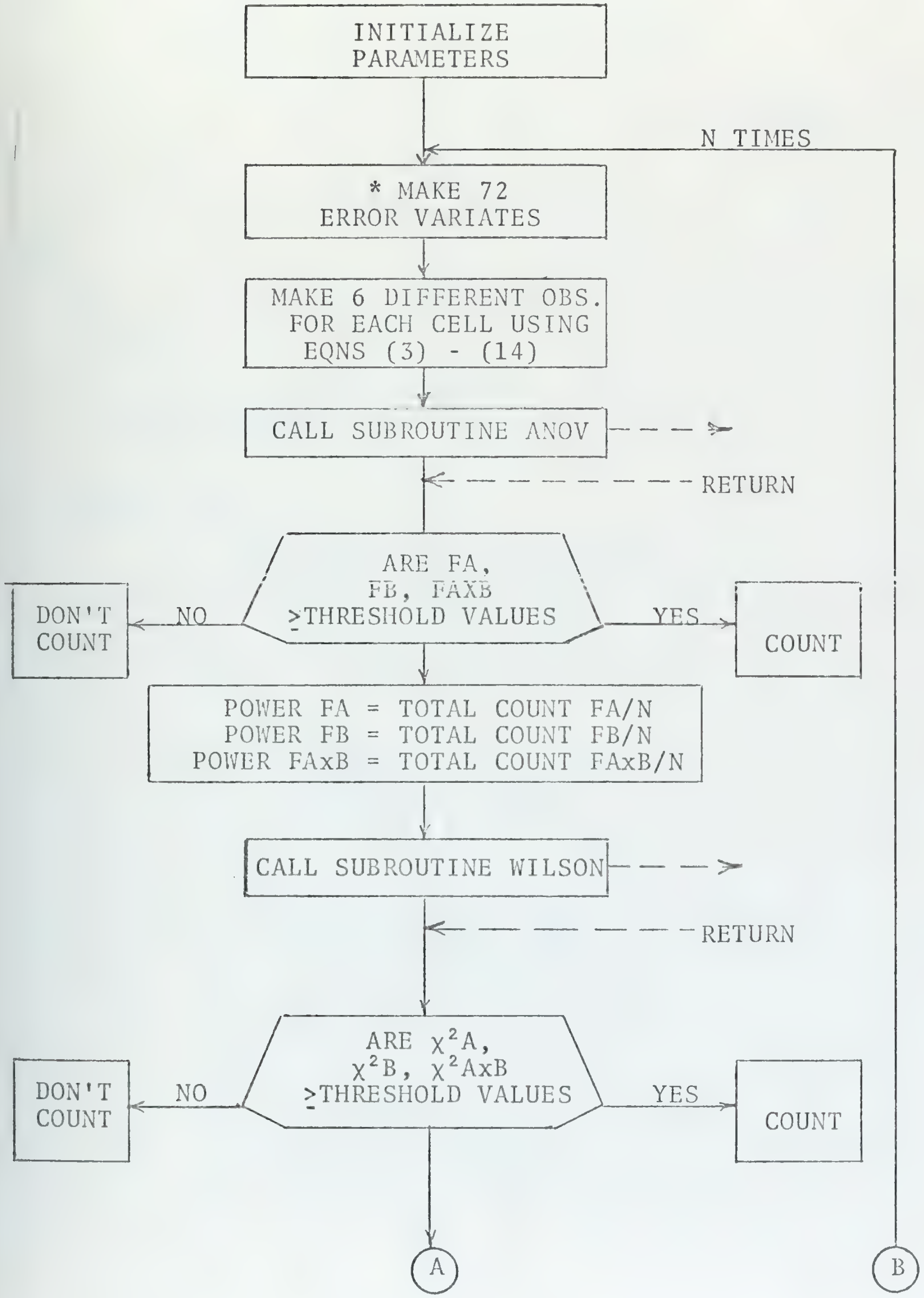
7. The overall mean, represented by μ in equations (3) through (14), was chosen arbitrarily to be 100.00.

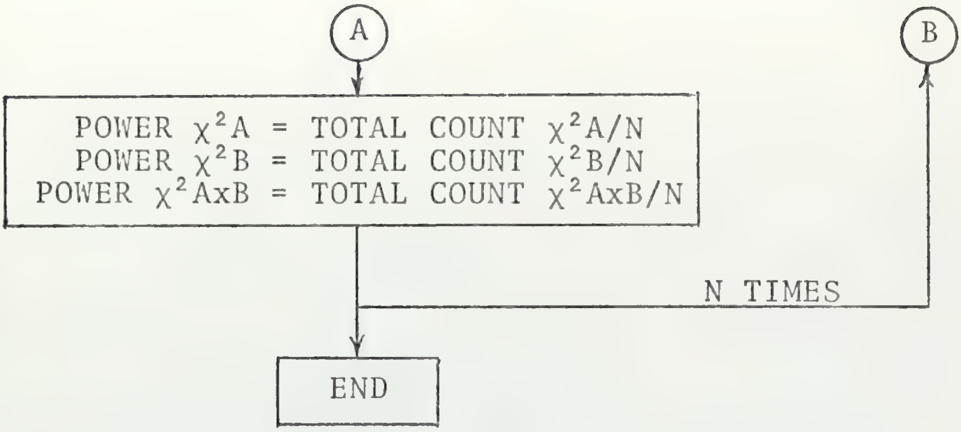
POWER OF THE ANALYSIS-OF-VARIANCE TEST



APPENDIX E: COMPUTER FLOW DIAGRAMS

1. Main Program


INITIALIZE PARAMETERS
N TIMES
* MAKE 72 ERROR VARIATES
MAKE 6 DIFFERENT OBS. FOR EACH CELL USING EQNS (3) - (14)
CALL SUBROUTINE ANOV
RETURN
ARE FA, FB, FAXB ≥THRESHOLD VALUES
NO
DON'T COUNT
YES
COUNT
POWER FA = TOTAL COUNT FA/N
POWER FB = TOTAL COUNT FB/N
POWER FAxB = TOTAL COUNT FAxB/N
CALL SUBROUTINE WILSON
RETURN
ARE χ²A, χ²B, χ²AxB ≥THRESHOLD VALUES
NO
DON'T COUNT
YES
COUNT
A
B

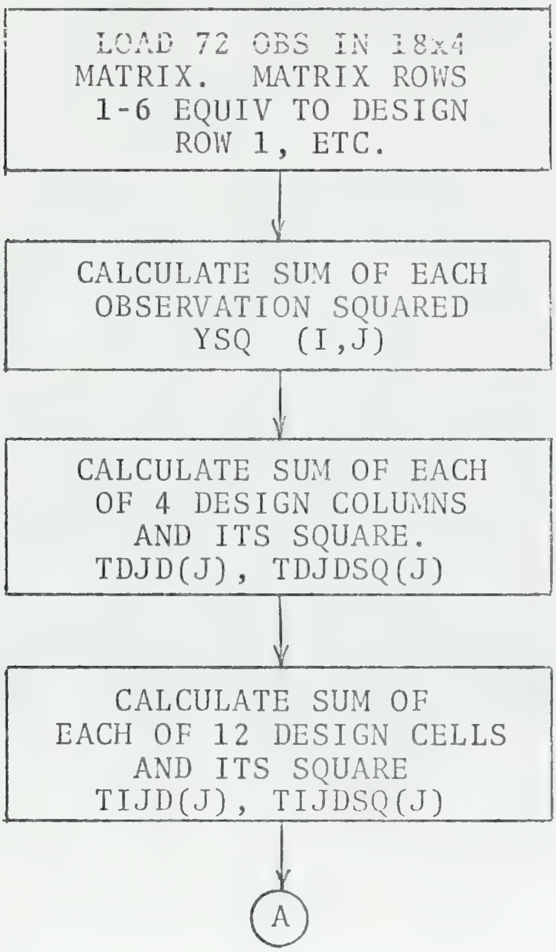

A
B
POWER χ^2A = TOTAL COUNT χ^2A/N
POWER χ^2B = TOTAL COUNT χ^2B/N
POWER χ^2AxB = TOTAL COUNT χ^2AxB/N
N TIMES
END

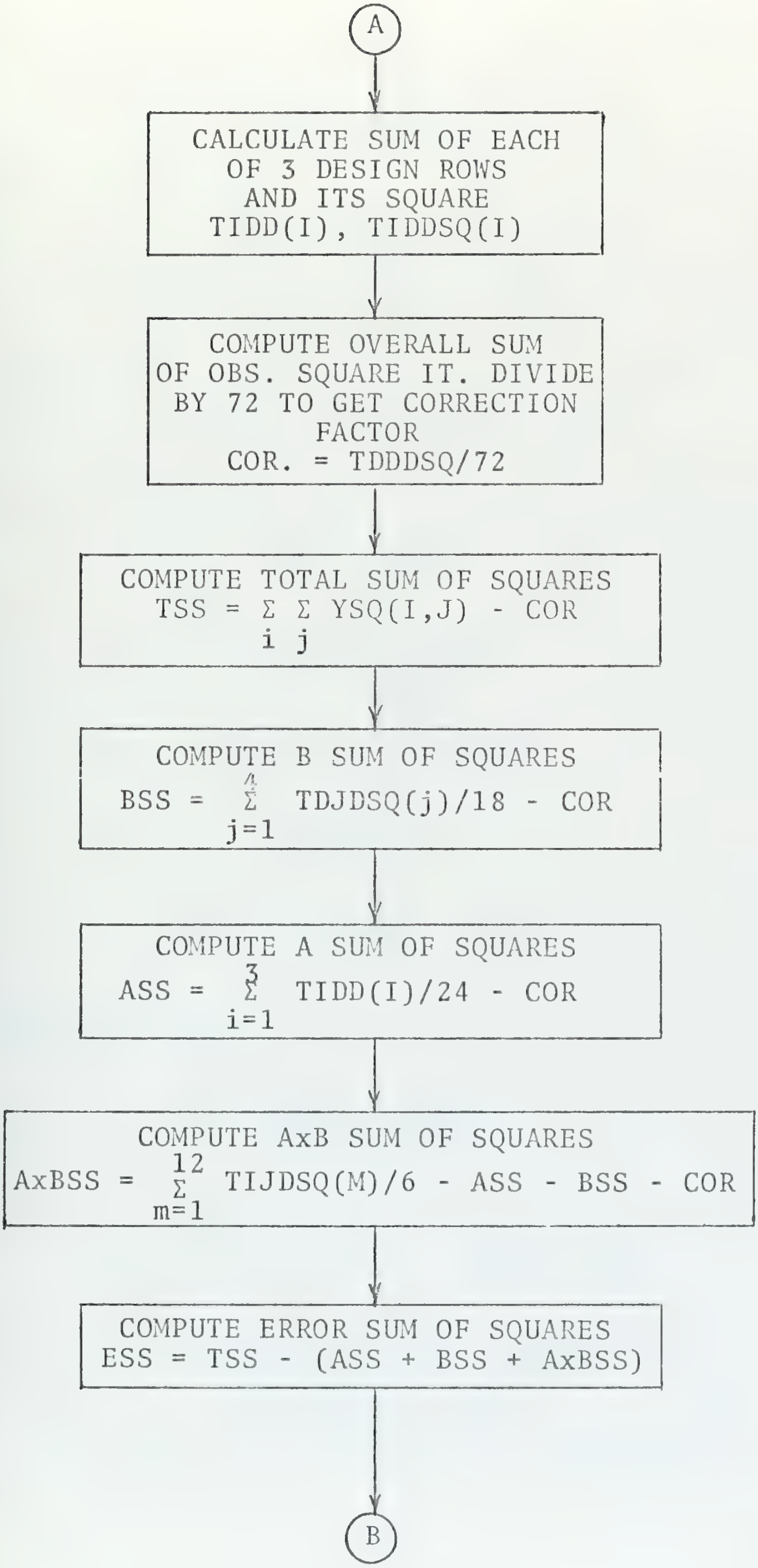
NOTES:

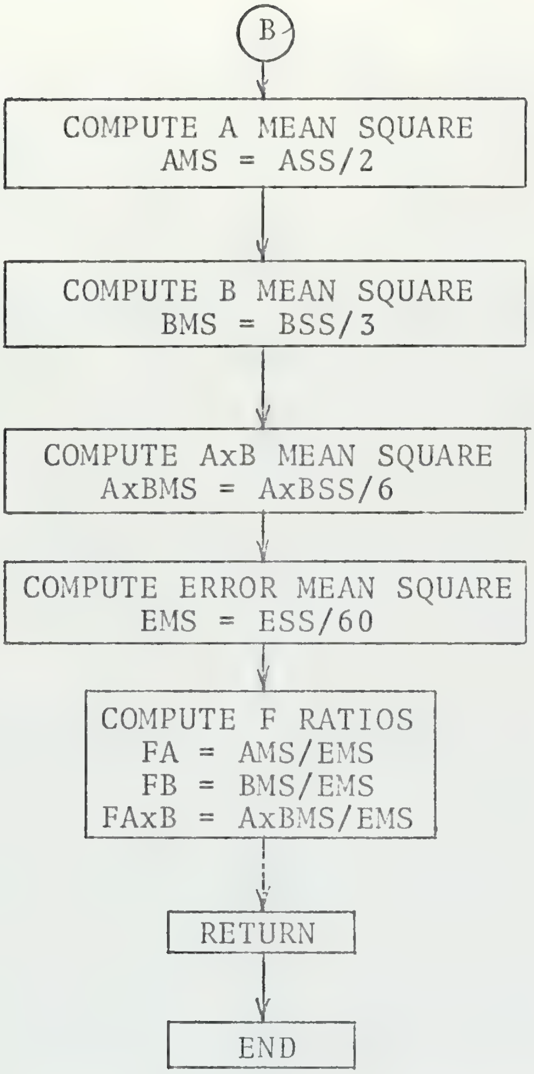
*This package was changed for each level of each type error.

Selected computer programs used are attached following appendices.

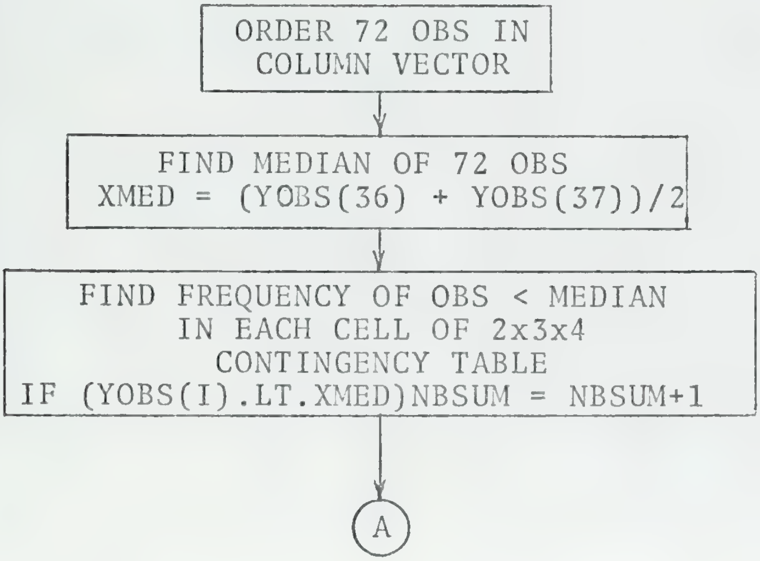
2. Subroutine ANOV

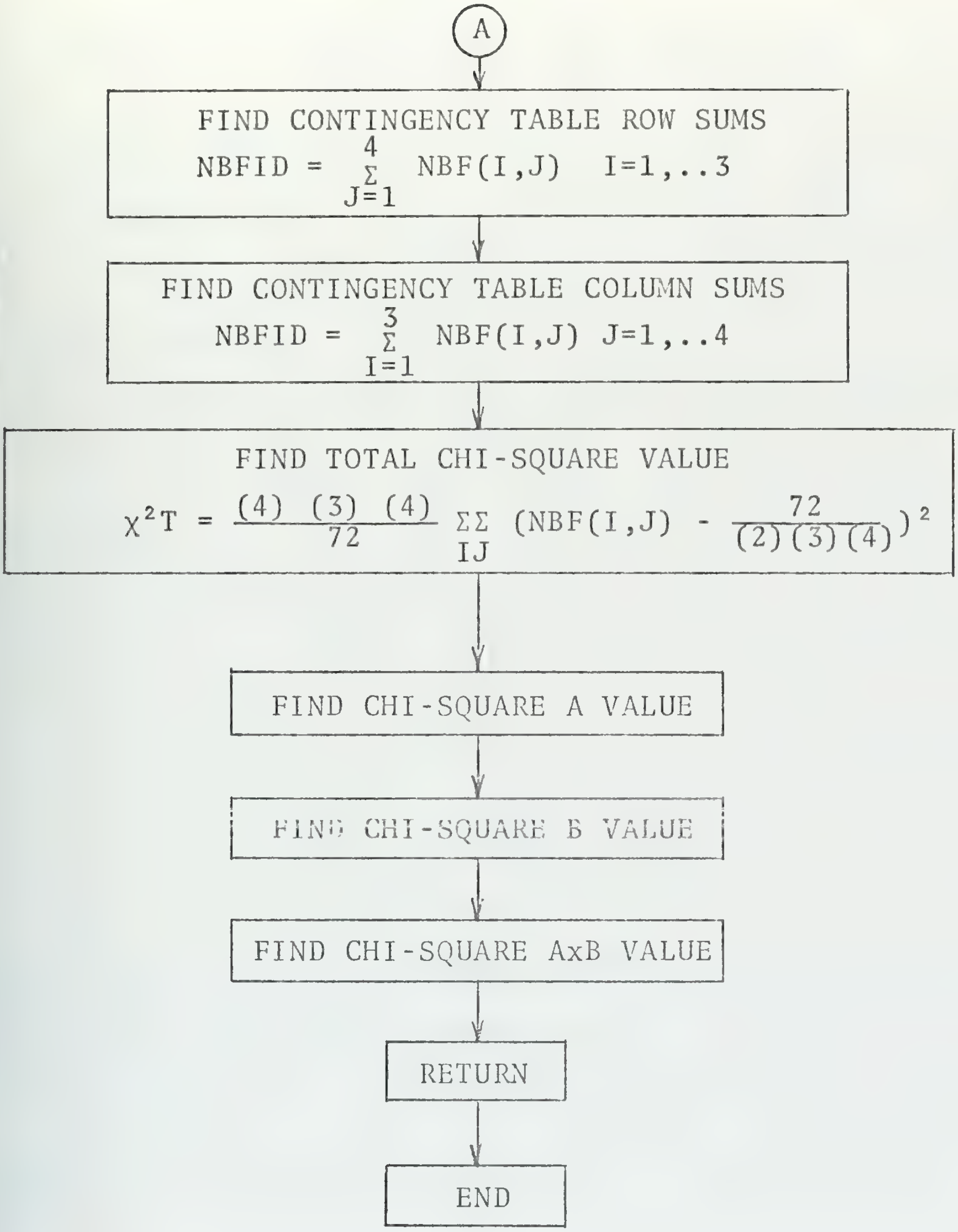

LOAD 72 OBS IN 18x4 MATRIX. MATRIX ROWS 1-6 EQUIV TO DESIGN ROW 1, ETC.
CALCULATE SUM OF EACH OBSERVATION SQUARED YSQ (I,J)
CALCULATE SUM OF EACH OF 4 DESIGN COLUMNS AND ITS SQUARE. TDJD(J), TDJDSQ(J)
CALCULATE SUM OF EACH OF 12 DESIGN CELLS AND ITS SQUARE TIJD(J), TIJDSQ(J)
A


A
CALCULATE SUM OF EACH OF 3 DESIGN ROWS AND ITS SQUARE
TIDD(I), TIDDSQ(I)
COMPUTE OVERALL SUM OF OBS. SQUARE IT. DIVIDE BY 72 TO GET CORRECTION FACTOR
COR. = TDDDSQ/72
COMPUTE TOTAL SUM OF SQUARES
$TSS = \sum_{i} \sum_{j} YSQ(I,J) - COR$
COMPUTE B SUM OF SQUARES
$BSS = \sum_{j=1}^{4} TDJDSQ(j)/18 - COR$
COMPUTE A SUM OF SQUARES
$ASS = \sum_{i=1}^{3} TIDD(I)/24 - COR$
COMPUTE AxB SUM OF SQUARES
$AxBSS = \sum_{m=1}^{12} TIJDSQ(M)/6 - ASS - BSS - COR$
COMPUTE ERROR SUM OF SQUARES
ESS = TSS - (ASS + BSS + AxBSS)
B



3. Subroutine Wilson


ORDER 72 OBS IN
COLUMN VECTOR
FIND MEDIAN OF 72 OBS
XMED = (YOBS(36) + YOBS(37))/2
FIND FREQUENCY OF OBS < MEDIAN
IN EACH CELL OF 2x3x4
CONTINGENCY TABLE
IF (YOBS(I).LT.XMED)NBSUM = NBSUM+1
A


A
FIND CONTINGENCY TABLE ROW SUMS
$NBFID = \sum_{J=1}^{4} NBF(I,J) \quad I=1,..3$
FIND CONTINGENCY TABLE COLUMN SUMS
$NBFID = \sum_{I=1}^{3} NBF(I,J) \quad J=1,..4$
FIND TOTAL CHI-SQUARE VALUE
$\chi^2 T = \frac{(4)\ (3)\ (4)}{72} \sum_{IJ}\sum (NBF(I,J) - \frac{72}{(2)(3)(4)})^2$
FIND CHI-SQUARE A VALUE
FIND CHI-SQUARE B VALUE
FIND CHI-SQUARE AxB VALUE
RETURN
END

APPENDIX F: DETERMINING TEST REPLICATIONS

1. Ten ANOV and ten WILSON subroutines were executed on data generated with the simulation model, N(0,4) error. Power of each of the F and Chi-square statistical tests for main effects and interaction was computed by the method shown in Appendix E. The subroutines were then executed ten more times and power computed in a similar manner. This was repeated until 200 ANOV's and WILSON subroutines had been executed consecutively giving a sample size of 20 power values per test, each power value based on the number of false null hypotheses rejected out of ten trials.

2. Sample variance was then computed for each test.

$$S^2 = \frac{1}{n_o - 1} \sum_{i=1}^{20} (X_i - \overline{X})^2 \qquad (6\text{-}1)$$

where X_i is the i^{th} power value and $\overline{X}$ is sample mean power value based on the 20 X_i's. $n_o = 20$.

3. Using the student's t distribution table, the value for k was determined so that the following equation held.

$$\text{Prob}\left[\frac{-e}{2\sqrt{k}} \leq t_{n_o - 1} \leq \frac{e}{2\sqrt{k}}\right] = 1-\alpha \qquad (6\text{-}2)$$

where $e = .04$, $n_o - 1 = 19$, and $1-\alpha = .95$.

4. For each of the six statistical tests involved $N_{\overline{X}}$, the number of replications required for desired confidence on the value of $\overline{X}$, was determined by:

$$N_{\overline{X}} = \mathrm{Max}\left(\left[\frac{S^2}{k}\right] + 1,\ n_o\right) \qquad (6\text{-}3)$$

where $[S^2/k]$ represents the greatest integer $\leq$ the ratio.

5. The value obtained for $N_{\overline{X}}$ was then multiplied by ten to obtain N, the number of WILSON and ANOV subroutines to be executed for desired confidence. The multiplication was necessary since ten replications were required for each value of X_i.

6. The maximum of the N values for the six tests involved was then determined and rounded up to the nearest 100. Results are summarized below:

$$\begin{array}{lllll} & \underline{A} & \underline{B} & \underline{AxB} & \\ N,\mathrm{ANOV} = \mathrm{Max} & (1560, & 520, & 1670) = 1670 & (6\text{-}4) \\ N,\mathrm{WILSON} - \mathrm{Max} & (3610, & 2930, & 1900) = 3610 & (6\text{-}5) \\ N = 3700 & & & & (6\text{-}6) \end{array}$$

7. Determining test was the Chi-square test for A effect. The computer program utilized is attached following the appendices.

APPENDIX G: THEORETICAL POWER, ANOV MAIN EFFECTS TEST

1. Reference 11 gives

$$\phi_A^2 = \frac{nc \sum_{i=1}^{r} \alpha_i^2}{\sigma^2 r} . \qquad (7\text{-}1)$$

Here n = number of replications, 6; c = number of design columns, 4; r = number of design rows, 3; σ^2 = desired variance, 4; and $\sum_i \alpha_i^2$ = sum of the squared α_i parameters from Appendix D. Substituting

$$\phi_A^2 = \frac{(6)(4)(1.7018)}{(4)(3)} = 3.40$$

and

$$\phi_A = 1.85.$$

The applicable power curve, attached to this appendix, has numerator degrees of freedom = r-1 = 2, and denominator degrees of freedom = rc(n-1) = 60. Power of the test, as read from the curve, is 0.81 at the 5 per cent level of significance.

2.

$$\phi_B^2 = \frac{nr \sum_{j=1}^{c} \beta_j^2}{\sigma^2 c} . \qquad (7\text{-}2)$$

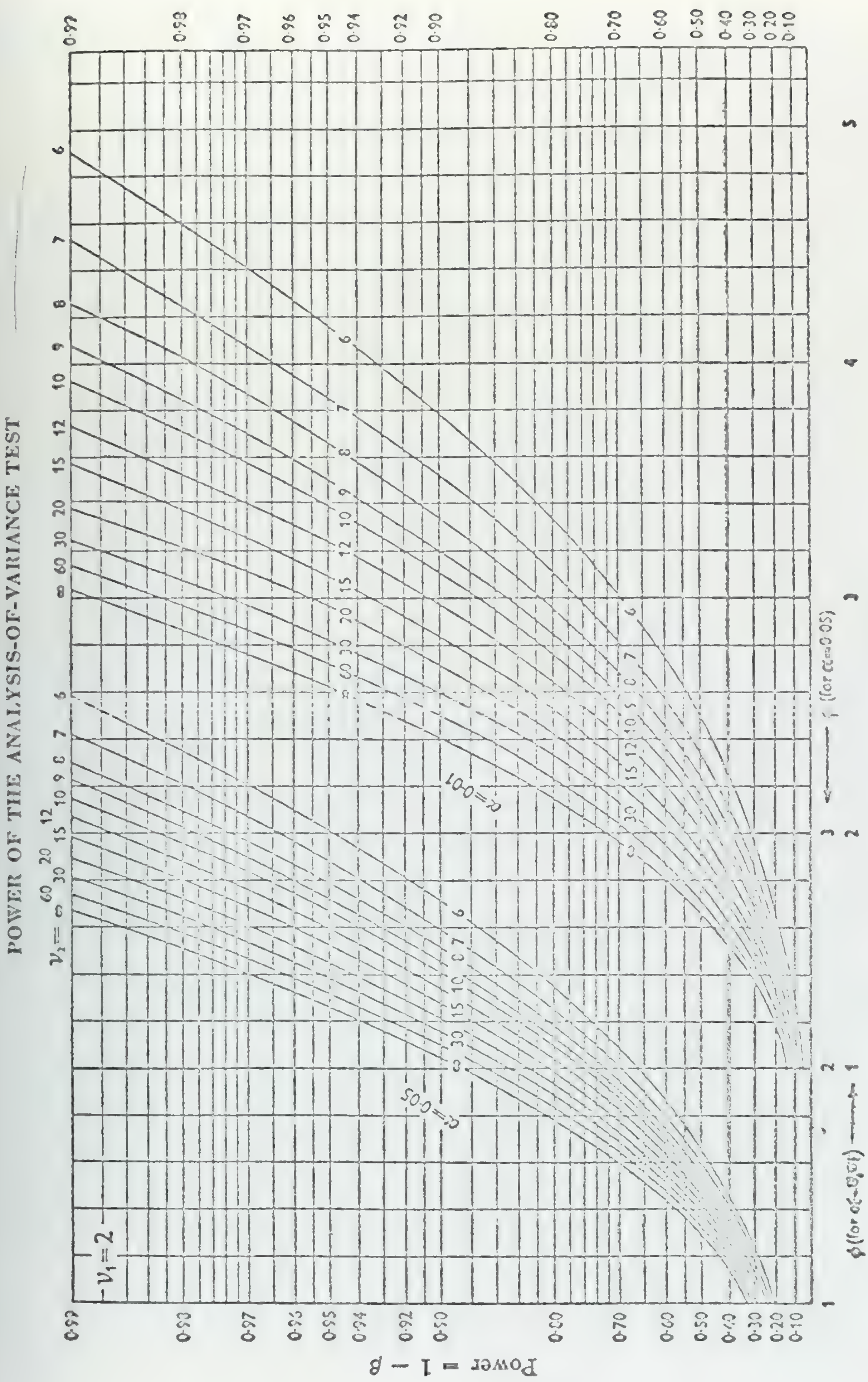
Here $\sum_j \beta_j^2$ = sum of the squared β_j's from Appendix D = 4.2436. Substituting

$$\phi_B^2 = \frac{(6)(3)(4.2436)}{(4)(4)} = 4.77$$

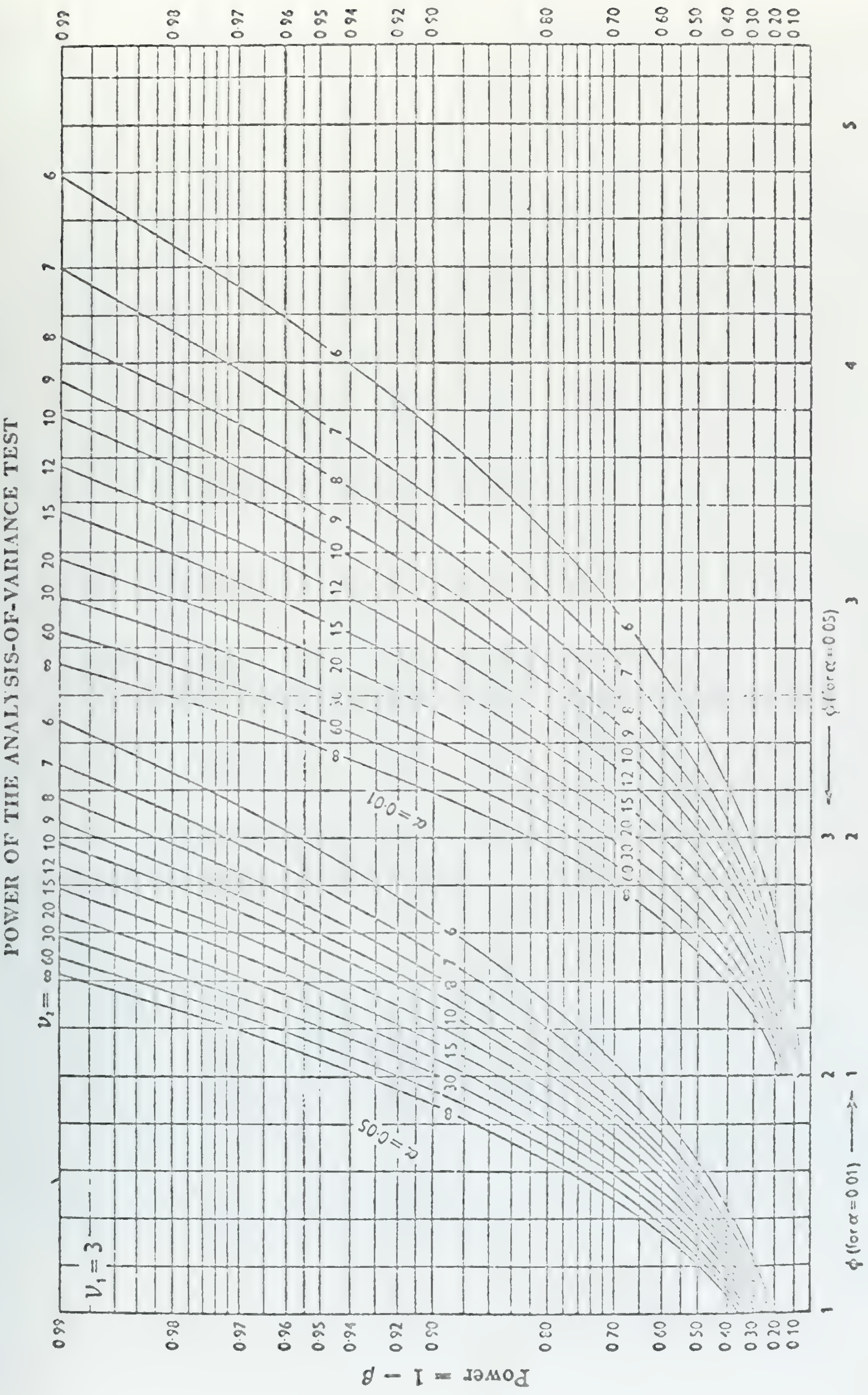
and

$$\phi_B = 2.18.$$

The applicable power curve, with numerator degrees of freedom = 3 and deonominator degrees of freedom = 60, gives power of this test = 0.96. The power curve is attached.



POWER OF THE ANALYSIS-OF-VARIANCE TEST



POWER OF THE ANALYSIS-OF-VARIANCE TEST

COMPUTER PROGRAM 1: DETERMINING TEST REPLICATIONS

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C        INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      SXIA = 0.0
      SXIB = 0.0
      SXIX = 0.0
      SXIASQ = 0.0
      SXIBSQ = 0.0
      SXIXSQ = 0.0
      SXIWA = 0.0
      SXIWB = 0.0
      SXIWX = 0.0
      SXWASQ = 0.0
      SXWBSQ = 0.0
      SXWXSQ = 0.0
      XL = 0.04
      TABVAL = 2.093
      NN = 20
      DO 1850 Z = 1,20
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      DO 1800 L = 1,10
      DO 50 I = 1,72
```

```
      SUM = 0.0
      DO 150 M = 1,12
      CALL RANDU (IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      SUM = SUM + RX(M)
  150 CONTINUE
      SDNORM(I) = 1.0*(SUM-6.0)+0.0
      YNORM(I) = SIGMA*SDNORM(I)
   50 CONTINUE
C         MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C         MODEL CELL 2
      DO 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
C         MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CONTINUE
C         MODEL CELL 4
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C         MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CONTINUE
C         MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CONTINUE
C         MODEL FOR CELL 7
      DO 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C         MODEL FOR CELL 8
      DO 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C         MODEL FOR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C         MODEL FOR CELL 10
      DO 1000 K = 55,60
```

```
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C         MODEL FOR CELL 11
      DC 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CONTINUE
C         MODEL FOR CELL 12
      DO 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
C         COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISWB,CSQAXB)
C         COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
 1800 CONTINUE
      XIA = NPASUM/10.0
      XIASQ = XIA**2
      XIB = NPBSUM/10.0
      XIBSQ = XIB**2
      XIX = NPXSUM/10.0
      XIXSQ = XIX**2
      XIWA = MPASUM/10.0
      XIWASQ = XIWA**2
      XIWB = MPBSUM/10.0
      XIWBSQ = XIWB**2
      XIWX = MPXSUM/10.0
      XIWXSQ = XIWX**2
      WRITE(6,4500) XIA,XIASQ,XIB,XIBSQ,XIX,XIXSQ
      WRITE(6,4500) XIWA,XIWASQ,XIWB,XIWBSQ,XIWX,XIWXSQ
 4500 FORMAT(1H,F8.4,3X,F8.4,5X,F8.4,3X,F8.4,5X,F8.4,3X,F8.4)
      SXIA = SXIA+XIA
      SXIB = SXIB+XIB
      SXIX = SXIX+XIX
      SXIASQ = SXIASQ+XIASQ
      SXIBSQ = SXIBSQ+XIBSQ
      SXIXSQ = SXIXSQ+XIXSQ
      SXIWA = SXIWA+XIWA
      SXIWB = SXIWB+XIWB
      SXIWX = SXIWX+XIWX
      SXWASQ = SXWASQ+XIWASQ
      SXWBSQ = SXWBSQ+XIWBSQ
```

```
      SXWXSQ = SXWXSQ+XIWXSQ
 1850 CONTINUE
      WRITE(6,4505) SXIA,SXIASQ,SXIB,SXIBSQ,SXIX,SXIXSQ
      WRITE(6,4505) SXIWA,SXWASQ,SXIWB,SXWBSQ,SXIWX,SXWXSQ
 4505 FORMAT(1H,F8.4,3X,F8.4,5X,F8.4,3X,F8.4,5X,F8.4,3X,F8.4)
      SQA = (SXIASQ-(SXIA**2/20.0))/19.0
      SQB = (SXIBSQ-(SXIB**2/20.0))/19.0
      SQX = (SXIXSQ-(SXIX**2/20.0))/19.0
      SQAW = (SXWASQ-(SXIWA**2/20.0))/19.0
      SQBW = (SXWBSQ-(SXIWB**2/20.0))/19.0
      SQXW = (SXWXSQ-(SXIWX**2/20.0))/19.0
      RTXZ = XL/(2*TABVAL)
      XZ = RTXZ**2
      RA = SQA/XZ
      NRA = IFIX(RA)
      RB = SQB/XZ
      NRB = IFIX(RB)
      RAXB = SQX/XZ
      NRX = IFIX(RAXB)
      RAW = SQAW/XZ
      NRAW = IFIX(RAW)
      RBW = SQBW/XZ
      NRBW = IFIX(RBW)
      RXW = SQXW/XZ
      NRXW = IFIX(RXW)
      NRA = NRA+1
      NRB = NRB+1
      NRX = NRX+1
      NRAW = NRAW+1
      NRBW = NRBW+1
      NRXW = NRXW+1
      WRITE(6,4510) NRA,NRB,NRX
      WRITE(6,4510) NRAW,NRBW,NRXW
 4510 FORMAT(1H,I6,5X,I6,5X,I6)
      NANOV = MAX0(NRA,NRB,NRX,NN)
      NWILS = MAX0(NRAW,NRBW,NRXW,NN)
      NINT = MAX0(NANOV,NWILS)
      NFIN = 10*NINT
      WRITE(6,4515) NANOV,NWILS,NINT,NFIN
 4515 FORMAT(1H,I6,5X,I6,5X,I6,5X,I6)
      STOP
      END
```

```
      SUBROUTINE ANOV(FA,FB,FAXB)
      DIMENSION WORK(18,4),YSQ(18,4),TDJD(10),TDJDSQ(10),TIJD(20),TIJDSQ
     1(20),TIDD(10),TIDDSQ(10)
      COMMON YOBS(100)
C        CONVERT VECTOR TO 18X4 MATRIX WITH CELLS IN CORRECT ORDER.
      DO 1900 I = 1,6
      WORK(I,1) = YOBS(I)
 1900 CONTINUE
      DO 2000 I = 1,6
      K = I+6
      WORK(I,2) = YOBS(K)
 2000 CONTINUE
      DO 2100 I = 1,6
      K = I+12
      WORK(I,3) = YOBS(K)
 2100 CONTINUE
      DO 2200 I = 1,6
      K = I+18
      WORK(I,4) = YOBS(K)
 2200 CONTINUE
      DO 2300 I = 7,12
      K = I+18
      WORK(I,1) = YOBS(K)
 2300 CONTINUE
      DO 2400 I = 7,12
      K = I+24
      WORK(I,2) = YOBS(K)
 2400 CONTINUE
      DO 2500 I = 7,12
      K = I+30
      WORK(I,3) = YOBS(K)
 2500 CONTINUE
      DO 2600 I = 7,12
      K = I+36
      WORK(I,4) = YOBS(K)
 2600 CONTINUE
      DO 2700 I = 13,18
      K = I+36
      WORK(I,1) = YOBS(K)
 2700 CONTINUE
      DO 2800 I = 13,18
      K = I+42
      WORK(I,2) = YOBS(K)
 2800 CONTINUE
      DO 2900 I = 13,18
      K = I+48
      WORK(I,3) = YOBS(K)
 2900 CONTINUE
```

```
      DO 3000 I = 13,18
      K = I+54
      WORK(I,4) = YOBS(K)
 3000 CONTINUE
C         FOR LATER FINDING TSS
      DO 3100 I = 1,18
      DO 3200 J = 1,4
      YSQ(I,J) = WORK(I,J)**2
 3200 CONTINUE
 3100 CONTINUE
C         FOR LATER FINDING BSS
      DO 3300 J = 1,4
      WSUM = 0.0
      DO 3400 I = 1,18
      WSUM = WSUM+WORK(I,J)
 3400 CONTINUE
      TDJD(J) = WSUM
      TDJDSQ(J) = TDJD(J)**2
 3300 CONTINUE
C         FOR LATER FINDING ASS AND AXBSS
      ISTART = 1
      IFIN = 6
      M = 0
 3550 DO 3500 K = 1,4
      ASUM = 0.0
      M = M+1
      DO 3600 I = ISTART,IFIN
      ASUM = ASUM+WORK(I,K)
 3600 CONTINUE
      TIJD(M) = ASUM
      TIJDSQ(M) = TIJD(M)**2
 3500 CONTINUE
      ISTART = ISTART+6
      IFIN = IFIN+6
      IF(ISTART.GT.13)GO TO 3700
      GO TO 3550
 3700 JSTART = 1
      JFIN = 4
      DO 3800 I = 1,3
      SSUM = 0.0
      DO 3900 K = JSTART,JFIN
      SSUM = SSUM+TIJD(K)
 3900 CONTINUE
      TIDD(I) = SSUM
      TIDDSQ(I) = TIDD(I)**2
      JSTART = JSTART+4
      JFIN = JFIN+4
      IF(JSTART.GT.9)GO TO 4000
```

```
 3800 CONTINUE
 4000 SUMYSQ = 0.0
C         COMPUTE SUMS OF SQUARES
      DO 4100 I = 1,18
      DO 4200 J = 1,4
      SUMYSQ = SUMYSQ+YSQ(I,J)
 4200 CONTINUE
 4100 CONTINUE
      TDDD = TDJD(1)+TDJD(2)+TDJD(3)+TDJD(4)
      TDDDSQ = TDDD**2
      COR = TDDDSQ/72
      TSS = SUMYSQ-COR
      BSS = (TDJDSQ(1)+TDJDSQ(2)+TDJDSQ(3)+TDJDSQ(4))/18 -COR
      ASS = (TIDDSQ(1)+TIDDSQ(2)+TIDDSQ(3))/24-COR
      STIJSQ = 0.0
      DO 4300  I = 1,12
      STIJSQ = STIJSQ+TIJDSQ(I)
 4300 CONTINUE
      AXBSS = STIJSQ/6-ASS-BSS-COR
      ESS = TSS-(ASS+BSS+AXBSS)
C         COMPUTE MEAN SQUARES
      AMS = ASS/2
      BMS = BSS/3
      AXBMS = AXBSS/6
      EMS = ESS/60
C         COMPUTE F RATIOS
      FA = AMS/EMS
      FB = BMS/EMS
      FAXB = AXBMS/EMS
      RETURN
      END


      SUBROUTINE WILSON(CHISQA,CHISQB,CSQAXB)
      DIMENSION NB(3,4),XOBS(100)
      COMMON YOBS(100)
      DO 6950 I = 1,72
      XOBS(I) = YOBS(I)
 6950 CONTINUE
C         ORDER OBSERVATIONS IN YOBS VECTOR
      LIM = 71
C         INITIALIZE INT TO 1 IN EVENT ALL YOBS(I) ARE IN ORDER
 7000 INT = 1
      DO 7100 I = 1,LIM
C         IF YOBS(I+1).LT.YOBS(I) INTERCHANGE THEM
      IF(YOBS(I+1).GE.YOBS(I)) GO TO 7100
      TEMP = YOBS(I+1)
      YOBS(I+1) = YOBS(I)
```

```
      YOBS(I) = TEMP
      INT = I
C        INT GIVES LOCATION OF LAST INTERCHANGE. ALL NUMBERS BEYOND
C        YOBS(INT) ARE IN ORDER.
 7100 CONTINUE
C        IF INT = 1,NO INTERCHANGES BEYOND YOBS(1) AND YOBS(2) HAVE
C        OCCURED. ALL NUMBERS ARE IN ORDER.
      IF(INT.EQ.1) GO TO 7200
      LIM = INT-1
      GO TO 7000
 7200 XMED = (YOBS(36)+YOBS(37))/2
      NBSUM = 0
      DO 7600 I = 1,6
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7600 CONTINUE
      NB(1,1) = NBSUM
      NBSUM = 0
      DO 7700 I = 7,12
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7700 CONTINUE
      NB(1,2) = NBSUM
      NBSUM = 0
      DO 7800 I = 13,18
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7800 CONTINUE
      NB(1,3) = NBSUM
      NBSUM = 0
      DO 7900 I = 19,24
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7900 CONTINUE
      NB(1,4) = NBSUM
      NBSUM = 0
      DO 8000 I = 25,30
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8000 CONTINUE
      NB(2,1) = NBSUM
      NBSUM = 0
      DO 8100 I = 31,36
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8100 CONTINUE
      NB(2,2) = NBSUM
      NBSUM = 0
      DO 8200 I = 37,42
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8200 CONTINUE
      NB(2,3) = NBSUM
      NBSUM = 0
      DO 8300 I = 43,48
```

```
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8300 CONTINUE
      NB(2,4) = NBSUM
      NBSUM = 0
      DO 8400 I = 49,54
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8400 CONTINUE
      NB(3,1) = NBSUM
      NBSUM = 0
      DO 8500 I = 55,60
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8500 CONTINUE
      NB(3,2) = NBSUM
      NBSUM = 0
      DO 8600 I = 61,66
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8600 CONTINUE
      NB(3,3) = NBSUM
      NBSUM = 0
      DO 8700 I = 67,72
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8700 CONTINUE
      NB(3,4) = NBSUM
      NBF1D = NB(1,1)+NB(1,2)+NB(1,3)+NB(1,4)
      NBF2D = NB(2,1)+NB(2,2)+NB(2,3)+NB(2,4)
      NBF3D = NB(3,1)+NB(3,2)+NB(3,3)+NB(3,4)
      NBFD1 = NB(1,1)+NB(2,1)+NB(3,1)
      NBFD2 = NB(1,2)+NB(2,2)+NB(3,2)
      NBFD3 = NB(1,3)+NB(2,3)+NB(3,3)
      NBFD4 = NB(1,4)+NB(2,4)+NB(3,4)
      NBDD = NBFD1+NBFD2+NBFD3+NBFD4
C        COMPUTATIONS FOR CHISQUARE TESTS
      TMULT = (4.0*3.0*4.0)/72.0
      NTCOR = 3
      CHISQT = TMULT*((NB(1,1)-3)**2+(NB(1,2)-3)**2+(NB(1,3)-3)**2+(NB(1
     1,4)-3)**2+(NB(2,1)-3)**2+(NB(2,2)-3)**2+(NB(2,3)-3)**2+(NB(2,4)-3)
     1**2+(NB(3,1)-3)**2+(NB(3,2)-3)**2+(NB(3,3)-3)**2+(NB(3,4)-3)**2)
      AMULT = (4.0*3.0)/72.0
      NACOR = 72/(2*3)
      CHISQA = AMULT*((NBF1D-NACOR)**2+(NBF2D-NACOR)**2+(NBF3D-NACOR)**2
     1)
      BMULT = (4.0*4.0)/72.0
      NBCOR = 72/(2*4)
      CHISQB = BMULT*((NBFD1-NBCOR)**2+(NBFD2-NBCOR)**2+(NBFD3-NBCOR)**2
     1+(NBFD4-NBCOR)**2)
      CSQAXB = CHISQT-CHISQA-CHISQB
      NADF = 2
      NBDF = 3
```

```
NAXBDF = 6
RETURN
END
```

COMPUTER PROGRAM 2: HISTOGRAM N(0,4) ERROR

```
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
      IX = 97531
      SIGMA = 2.0
      EX = 0.0
      SUMNRM = 0.0
      SNORSQ = 0.0
      SNORCU = 0.0
      SNORFR = 0.0
      N = 266400
      DO 1800 L = 1,3700
      DO 50 I = 1,72
      SUM = 0.0
      DO 100 M = 1,12
      CALL RANDU(IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      SUM = SUM+RX(M)
  100 CONTINUE
      SDNORM(I) = 1.0*(SUM-6.0)+0.0
      YNORM(I) = EX+SIGMA*SDNORM(I)
  325 SUMNRM = SUMNRM+YNORM(I)
      YNORSQ(I) = YNORM(I)**2
      SNORSQ = SNORSQ + YNORSQ(I)
      YNORCU(I) = YNORSQ(I)*YNORM(I)
      YNORFR(I) = YNORSQ(I)**2
      SNORCU = SNORCU+YNORCU(I)
      SNORFR = SNORFR+YNORFR(I)
      IF(YNORM(I).GE.12.0) GO TO 41
      IF(YNORM(I).LE.-12.0) GO TO 22
      Y = ((YNORM(I)+12.0)*20.0)/24.0
      K = Y + 1
      GO TO (22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41
     1),K
   22 NBOX(1) = NBOX(1) + 1
      GO TO 50
   23 NBOX(2) = NBOX(2) + 1
      GO TO 50
   24 NBOX(3) = NBOX(3) + 1
      GO TO 50
   25 NBOX(4) = NBOX(4) + 1
      GO TO 50
   26 NBOX(5) = NBOX(5) + 1
```

```
      GO TO 50
   27 NBOX(6) = NBOX(6) + 1
      GO TO 50
   28 NBOX(7) = NBOX(7) + 1
      GO TO 50
   29 NBOX(8) = NBOX(8) + 1
      GO TO 50
   30 NBOX(9) = NBOX(9) + 1
      GO TO 50
   31 NBOX(10) = NBOX(10) + 1
      GO TO 50
   32 NBOX(11) = NBOX(11) + 1
      GO TO 50
   33 NBOX(12) = NBOX(12) + 1
      GO TO 50
   34 NBOX(13) = NBOX(13) + 1
      GO TO 50
   35 NBOX(14) = NBOX(14) + 1
      GO TO 50
   36 NBOX(15) = NBOX(15) + 1
      GO TO 50
   37 NBOX(16) = NBOX(16) + 1
      GO TO 50
   38 NBOX(17) = NBOX(17) + 1
      GO TO 50
   39 NBOX(18) = NBOX(18) + 1
      GO TO 50
   40 NBOX(19) = NBOX(19) + 1
      GO TO 50
   41 NBOX(20) = NBOX(20) + 1
   50 CONTINUE
 1800 CONTINUE
      YNMEAN = SUMNRM/N
      YNVAR = (SNORSQ - (SUMNRM**2/N))/(N - 1)
      YMOM3 = (SNORCU-(3.0*SUMNRM*SNORSQ)/N+((SUMNRM**2)*3.0*SUMNRM)/(N*
     1*2)-(SUMNRM**3)/(N**2))/(N-1)
      YMOM4 = (SNORFR-(4.0*SUMNRM*SNORCU)/N+((SUMNRM**2)*6.0*SNORSQ)/(N*
     1*2)-((SUMNRM**3)*4.0*SUMNRM)/(N**3)+(SUMNRM**4)/(N**3))/(N-1)
      BETA1 = (YMOM3**2)/(YNVAR**3)
      SBETA1 = SQRT(BETA1)
      BETA2 = YMOM4/(YNVAR**2)
      DO 3000 I = 1,20
      WRITE (6,2500)I,YNMEAN,YNVAR,NBOX(I)
 2500 FORMAT(1H,I2,10X,2F15.3,5X,I6)
 3000 CONTINUE
      WRITE(6,9500) YMOM3,YMOM4
 9500 FORMAT(1H,2F25.5)
      WRITE(6,9600) SBETA1,BETA1,BETA2
```

```
9600 FCRMAT(1H,3F15.5)
     DO 4000 J = 1,72
     WRITE(6,3500) J,YNORM(J)
3500 FCRMAT(1H,I2,10X,F15.5)
4000 CCNTINUE
     STOP
     END
```

COMPUTER PROGRAM 3: POWER N(0,4) ERROR (VALIDATION)

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C         INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      N = 3700
      DO 1800 L = 1,N
      DO 50 I = 1,72
      SUM = 0.0
      DO 150 M = 1,12
      CALL RANDU (IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      SUM = SUM + RX(M)
  150 CONTINUE
      SDNORM(I) = 1.0*(SUM-6.0)+0.0
      YNORM(I) = SIGMA*SDNORM(I)
   50 CONTINUE
C         MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C         MODEL CELL 2
```

```
      DC 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
C        MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CCNTINUE
C        MODEL CELL 4
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CCNTINUE
C        MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CCNTINUE
C        MODEL CELL 6
      DC 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CCNTINUE
C        MODEL FOR CELL 7
      DC 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C        MODEL FOR CELL 8
      DC 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CCNTINUE
C        MODEL FCR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CCNTINUE
C        MODEL FOR CELL 10
      DC 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C        MODEL FOR CELL 11
      DC 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CCNTINUE
C        MODEL FOR CELL 12
      DC 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CCNTINUE
      CALL ANOV(FA,FB,FAXB)
C        CCMPARE AND TOTAL FCR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
```

```
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISQB,CSQAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
 1800 CONTINUE
      XPOWA = NPASUM*1.0/(1.0*N)
      XPOWB = NPBSUM*1.0/(1.0*N)
      XPOWX = NPXSUM*1.0/(1.0*N)
      XPOWAW = MPASUM*1.0/(1.0*N)
      XPOWBW = MPBSUM*1.0/(1.0*N)
      XPOWXW = MPXSUM*1.0/(1.0*N)
      WRITE(6,4500)XPOWA,XPOWB,XPOWX
      WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
 4500 FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
      STOP
      END


      SUBROUTINE ANOV(FA,FB,FAXB)
      DIMENSION WORK(18,4),YSQ(18,4),TDJD(10),TDJDSQ(10),TIJD(20),TIJDSQ
     1(20),TIDD(10),TIDDSQ(10)
      COMMON YOBS(100)
C        CONVERT VECTOR TO 18X4 MATRIX WITH CELLS IN CORRECT ORDER.
      DO 1900 I = 1,6
      WORK(I,1) = YOBS(I)
 1900 CONTINUE
      DO 2000 I = 1,6
      K = I+6
      WORK(I,2) = YOBS(K)
 2000 CONTINUE
      DO 2100 I = 1,6
      K = I+12
      WORK(I,3) = YOBS(K)
 2100 CONTINUE
      DO 2200 I = 1,6
      K = I+18
      WORK(I,4) = YOBS(K)
 2200 CONTINUE
      DO 2300 I = 7,12
      K = I+18
      WORK(I,1) = YOBS(K)
 2300 CONTINUE
      DO 2400 I = 7,12
      K = I+24
      WORK(I,2) = YOBS(K)
 2400 CONTINUE
```

```
      DO 2500 I = 7,12
      K = I+30
      WORK(I,3) = YOBS(K)
 2500 CONTINUE
      DC 2600 I = 7,12
      K = I+36
      WORK(I,4) = YOBS(K)
 2600 CCNTINUE
      DC 2700 I = 13,18
      K = I+36
      WORK(I,1) = YOBS(K)
 2700 CCNTINUE
      DO 2800 I = 13,18
      K = I+42
      WORK(I,2) = YOBS(K)
 2800 CONTINUE
      CC 2900 I = 13,18
      K = I+48
      WORK(I,3) = YOBS(K)
 2900 CCNTINUE
      DC 3000 I = 13,18
      K = I+54
      WORK(I,4) = YOBS(K)
 3000 CCNTINUE
C        FOR LATER FINDING TSS
      DO 3100 I = 1,18
      DO 3200 J = 1,4
      YSQ(I,J) = WORK(I,J)**2
 3200 CCNTINUE
 3100 CONTINUE
C        FOR LATER FINDING BSS
      DC 3300 J = 1,4
      WSUM = 0.0
      DC 3400 I = 1,18
      WSUM = WSUM+WORK(I,J)
 3400 CCNTINUE
      TDJD(J) = WSUM
      TDJDSQ(J) = TDJD(J)**2
 3300 CONTINUE
C        FOR LATER FINDING ASS AND AXBSS
      ISTART = 1
      IFIN = 6
      M = 0
 3550 DC 3500 K = 1,4
      ASUM = 0.0
      M = M+1
      DC 3600 I = ISTART,IFIN
      ASUM = ASUM+WORK(I,K)
```

```
 3600 CONTINUE
      TIJD(M) = ASUM
      TIJDSQ(M) = TIJD(M)**2
 3500 CONTINUE
      ISTART = ISTART+6
      IFIN = IFIN+6
      IF(ISTART.GT.13)GO TO 3700
      GO TO 3550
 3700 JSTART = 1
      JFIN = 4
      DO 3800 I = 1,3
      SSUM = 0.0
      DO 3900 K = JSTART,JFIN
      SSUM = SSUM+TIJD(K)
 3900 CONTINUE
      TIDD(I) = SSUM
      TIDDSQ(I) = TIDD(I)**2
      JSTART = JSTART+4
      JFIN = JFIN+4
      IF(JSTART.GT.9)GO TO 4000
 3800 CONTINUE
 4000 SUMYSQ = 0.0
C        COMPUTE SUMS OF SQUARES
      DO 4100 I = 1,18
      DO 4200 J = 1,4
      SUMYSQ = SUMYSQ+YSQ(I,J)
 4200 CONTINUE
 4100 CONTINUE
      TDDD = TDJD(1)+TDJD(2)+TDJD(3)+TDJD(4)
      TDDDSQ = TDDD**2
      COR = TDDDSQ/72
      TSS = SUMYSQ-COR
      BSS = (TDJDSQ(1)+TDJDSQ(2)+TDJDSQ(3)+TDJDSQ(4))/18 -COR
      ASS = (TIDDSQ(1)+TIDDSQ(2)+TIDDSQ(3))/24-COR
      STIJSQ = 0.0
      DO 4300 I = 1,12
      STIJSQ = STIJSQ+TIJDSQ(I)
 4300 CONTINUE
      AXBSS = STIJSQ/6-ASS-BSS-COR
      ESS = TSS-(ASS+BSS+AXBSS)
C        COMPUTE MEAN SQUARES
      AMS = ASS/2
      BMS = BSS/3
      AXBMS = AXBSS/6
      EMS = ESS/60
C        COMPUTE F RATIOS
      FA = AMS/EMS
      FB = BMS/EMS
```

```
      FAXB = AXBMS/EMS
      RETURN
      END

      SUBROUTINE WILSON(CHISQA,CHISQB,CSQAXB)
      DIMENSION NB(3,4),XOBS(100)
      COMMON YOBS(100)
      DO 6950 I = 1,72
      XOBS(I) = YOBS(I)
 6950 CONTINUE
C        ORDER OBSERVATIONS IN YOBS VECTOR
      LIM = 71
C        INITIALIZE INT TO 1 IN EVENT ALL YOBS(I) ARE IN ORDER
 7000 INT = 1
      DO 7100 I = 1,LIM
C        IF YOBS(I+1).LT.YOBS(I) INTERCHANGE THEM
      IF(YOBS(I+1).GE.YOBS(I)) GO TO 7100
      TEMP = YOBS(I+1)
      YOBS(I+1) = YOBS(I)
      YOBS(I) = TEMP
      INT = I
C        INT GIVES LOCATION OF LAST INTERCHANGE. ALL NUMBERS BEYOND
C        YOBS(INT) ARE IN ORDER.
 7100 CONTINUE
C        IF INT = 1,NO INTERCHANGES BEYOND YOBS(1) AND YOBS(2) HAVE
C        OCCURED. ALL NUMBERS ARE IN ORDER.
      IF(INT.EQ.1) GO TO 7200
      LIM = INT-1
      GO TO 7000
 7200 XMED = (YOBS(36)+YOBS(37))/2
      NBSUM = 0
      DO 7600 I = 1,6
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7600 CONTINUE
      NB(1,1) = NBSUM
      NBSUM = 0
      DO 7700 I = 7,12
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7700 CONTINUE
      NB(1,2) = NBSUM
      NBSUM = 0
      DO 7800 I = 13,18
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7800 CONTINUE
      NB(1,3) = NBSUM
      NBSUM = 0
      DO 7900 I = 19,24
```

```
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7900 CONTINUE
      NB(1,4) = NBSUM
      NBSUM = 0
      DO 8000 I = 25,30
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8000 CONTINUE
      NB(2,1) = NBSUM
      NBSUM = 0
      DO 8100 I = 31,36
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8100 CONTINUE
      NB(2,2) = NBSUM
      NBSUM = 0
      DO 8200 I = 37,42
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8200 CONTINUE
      NB(2,3) = NBSUM
      NBSUM = 0
      DO 8300 I = 43,48
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8300 CONTINUE
      NB(2,4) = NBSUM
      NBSUM = 0
      DO 8400 I = 49,54
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8400 CONTINUE
      NB(3,1) = NBSUM
      NBSUM = 0
      DO 8500 I = 55,60
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8500 CONTINUE
      NB(3,2) = NBSUM
      NBSUM = 0
      DO 8600 I = 61,66
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8600 CONTINUE
      NB(3,3) = NBSUM
      NBSUM = 0
      DO 8700 I = 67,72
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8700 CONTINUE
      NB(3,4) = NBSUM
      NBF1D = NB(1,1)+NB(1,2)+NB(1,3)+NB(1,4)
      NBF2D = NB(2,1)+NB(2,2)+NB(2,3)+NB(2,4)
      NBF3D = NB(3,1)+NB(3,2)+NB(3,3)+NB(3,4)
      NBFD1 = NB(1,1)+NB(2,1)+NB(3,1)
      NBFD2 = NB(1,2)+NB(2,2)+NB(3,2)
```

```
      NBFD3 = NB(1,3)+NB(2,3)+NB(3,3)
      NBFD4 = NB(1,4)+NB(2,4)+NB(3,4)
      NBDD = NBFD1+NBFD2+NBFD3+NBFD4
C        COMPUTATIONS FOR CHISQUARE TESTS
      TMULT = (4.0*3.0*4.0)/72.0
      NTCOR = 3
      CHISQT = TMULT*((NB(1,1)-3)**2+(NB(1,2)-3)**2+(NB(1,3)-3)**2+(NB(1
     1,4)-3)**2+(NB(2,1)-3)**2+(NB(2,2)-3)**2+(NB(2,3)-3)**2+(NB(2,4)-3)
     1**2+(NB(3,1)-3)**2+(NB(3,2)-3)**2+(NB(3,3)-3)**2+(NB(3,4)-3)**2)
      AMULT = (4.0*3.0)/72.0
      NACOR = 72/(2*3)
      CHISQA = AMULT*((NBF1D-NACOR)**2+(NBF2D-NACOR)**2+(NBF3D-NACOR)**2
     1)
      BMULT = (4.0*4.0)/72.0
      NBCOR = 72/(2*4)
      CHISQB = BMULT*((NBFD1-NBCOR)**2+(NBFD2-NBCOR)**2+(NBFD3-NBCOR)**2
     1+(NBFD4-NBCOR)**2)
      CSQAXB = CHISQT-CHISQA-CHISQB
      NADF = 2
      NBDF = 3
      NAXBDF = 6
      RETURN
      END
```

COMPUTER PROGRAM 4: LEVELS OF SIGNIFICANCE (VALIDATION)

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C         INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = 0.0
      A2 = 0.0
      B1 = 0.0
      B2 = 0.0
      B3 = 0.0
      AB11 = 0.0
      AB12 = 0.0
      AB13 = 0.0
      AB21 = 0.0
      AB22 = 0.0
      AB23 = 0.0
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      JPASUM = 0
      JPBSUM = 0
      JPXSUM = 0
      KPASUM = 0
      KPBSUM = 0
      KPXSUM = 0
      N = 3700
      DO 1800 L = 1,N
      DO 50 I = 1,72
      SUM = 0.0
      DO 150 M = 1,12
      CALL RANDU (IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      SUM = SUM + RX(M)
  150 CONTINUE
      SDNORM(I) = 1.0*(SUM-6.0)+0.0
      YNORM(I) = SIGMA*SDNORM(I)
```

```
   50 CCNTINUE
C        MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CCNTINUE
C        MODEL CELL 2
      DC 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CCNTINUE
C        MODEL CELL 3
      DC 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CCNTINUE
C        MODEL CELL 4
      DC 400 K = 19,24
      YCBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C        MODEL CELL 5
      DC 500 K = 25,30
      YCBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CCNTINUE
C        MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CCNTINUE
C        MODEL FOR CELL 7
      DC 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CCNTINUE
C        MODEL FOR CELL 8
      DC 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C        MODEL FOR CELL 9
      DC 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C        MODEL FOR CELL 10
      DC 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CCNTINUE
C        MODEL FCR CELL 11
      DO 1100 K = 61,66
      YCBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CCNTINUE
C        MODEL FOR CELL 12
      DC 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
```

```
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      IF(FA.GE.2.39)JPASUM = JPASUM+1
      IF(FB.GE.2.18)JPBSUM = JPBSUM+1
      IF(FAXB.GE.1.87)JPXSUM = JPXSUM+1
      CALL WILSON(CHISCA,CHISQB,CSQAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
      IF(CHISQA.GE.4.61)KPASUM = KPASUM+1
      IF(CHISQB.GE.6.25)KPBSUM = KPBSUM+1
      IF(CSQAXB.GE.10.6)KPXSUM = KPXSUM+1
 1800 CONTINUE
      XPOWA = NPASUM*1.0/(1.0*N)
      XPOWB = NPBSUM*1.0/(1.0*N)
      XPOWX = NPXSUM*1.0/(1.0*N)
      XPOWAW = MPASUM*1.0/(1.0*N)
      XPOWBW = MPBSUM*1.0/(1.0*N)
      XPOWXW = MPXSUM*1.0/(1.0*N)
      XPOWA1 = JPASUM*1.0/(1.0*N)
      XPOWB1 = JPBSUM*1.0/(1.0*N)
      XPOWX1 = JPXSUM*1.0/(1.0*N)
      WPOWA1 = KPASUM*1.0/(1.0*N)
      WPOWB1 = KPBSUM*1.0/(1.0*N)
      WPOWX1 = KPXSUM*1.0/(1.0*N)
      WRITE(6,4500)XPOWA,XPOWB,XPOWX
      WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
      WRITE(6,4500) XPOWA1,XPOWB1,XPOWX1
      WRITE(6,4500) WPOWA1,WPOWB1,WPOWX1
 4500 FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
      STOP
      END


      SUBROUTINE ANOV(FA,FB,FAXB)
      DIMENSION WORK(18,4),YSQ(18,4),TDJD(10),TDJDSQ(10),TIJD(20),TIJDSQ
     1(20),TIDD(10),TIDDSQ(10)
      COMMON YOBS(100)
C        CONVERT VECTOR TO 18X4 MATRIX WITH CELLS IN CORRECT ORDER.
      DO 1900 I = 1,6
      WORK(I,1) = YOBS(I)
 1900 CONTINUE
```

```
      DO 2000 I = 1,6
      K = I+6
      WORK(I,2) = YOBS(K)
 2000 CONTINUE
      DO 2100 I = 1,6
      K = I+12
      WORK(I,3) = YOBS(K)
 2100 CONTINUE
      DO 2200 I = 1,6
      K = I+18
      WORK(I,4) = YOBS(K)
 2200 CONTINUE
      DO 2300 I = 7,12
      K = I+18
      WORK(I,1) = YOBS(K)
 2300 CONTINUE
      DO 2400 I = 7,12
      K = I+24
      WORK(I,2) = YOBS(K)
 2400 CONTINUE
      DO 2500 I = 7,12
      K = I+30
      WORK(I,3) = YOBS(K)
 2500 CONTINUE
      DO 2600 I = 7,12
      K = I+36
      WORK(I,4) = YOBS(K)
 2600 CONTINUE
      DO 2700 I = 13,18
      K = I+26
      WORK(I,1) = YOBS(K)
 2700 CONTINUE
      DO 2800 I = 13,18
      K = I+42
      WORK(I,2) = YOBS(K)
 2800 CONTINUE
      DO 2900 I = 13,18
      K = I+48
      WORK(I,3) = YOBS(K)
 2900 CONTINUE
      DO 3000 I = 13,18
      K = I+54
      WORK(I,4) = YOBS(K)
 3000 CONTINUE
C        FOR LATER FINDING TSS
      DO 3100 I = 1,18
      DO 3200 J = 1,4
      YSQ(I,J) = WORK(I,J)**2
```

```
 3200 CONTINUE
 3100 CONTINUE
C        FOR LATER FINDING BSS
      DO 3300 J = 1,4
      WSUM = 0.0
      DO 3400 I = 1,18
      WSUM = WSUM+WORK(I,J)
 3400 CONTINUE
      TDJD(J) = WSUM
      TDJDSQ(J) = TDJD(J)**2
 3300 CONTINUE
C        FOR LATER FINDING ASS AND AXBSS
      ISTART = 1
      IFIN = 6
      M = 0
 3550 DO 3500 K = 1,4
      ASUM = 0.0
      M = M+1
      DO 3600 I = ISTART,IFIN
      ASUM = ASUM+WORK(I,K)
 3600 CONTINUE
      TIJD(M) = ASUM
      TIJDSQ(M) = TIJD(M)**2
 3500 CONTINUE
      ISTART = ISTART+6
      IFIN = IFIN+6
      IF(ISTART.GT.13)GO TO 3700
      GO TO 3550
 3700 JSTART = 1
      JFIN = 4
      DO 3800 I = 1,3
      SSUM = 0.0
      DO 3900 K = JSTART,JFIN
      SSUM = SSUM+TIJD(K)
 3900 CONTINUE
      TIDD(I) = SSUM
      TIDDSQ(I) = TIDD(I)**2
      JSTART = JSTART+4
      JFIN = JFIN+4
      IF(JSTART.GT.9)GO TO 4000
 3800 CONTINUE
 4000 SUMYSQ = 0.0
C        COMPUTE SUMS OF SQUARES
      DO 4100 I = 1,18
      DO 4200 J = 1,4
      SUMYSQ = SUMYSQ+YSQ(I,J)
 4200 CONTINUE
 4100 CONTINUE
```

```
      TDDD = TDJD(1)+TDJD(2)+TDJD(3)+TDJD(4)
      TDDDSQ = TDDD**2
      COR = TDDDSQ/72
      TSS = SUMYSQ-COR
      BSS = (TDJDSQ(1)+TDJDSQ(2)+TDJDSQ(3)+TDJDSQ(4))/18 -COR
      ASS = (TIDDSQ(1)+TIDDSQ(2)+TIDDSQ(3))/24-COR
      STIJSQ = 0.0
      DO 4300 I = 1,12
      STIJSQ = STIJSQ+TIJDSQ(I)
 4300 CONTINUE
      AXBSS = STIJSQ/6-ASS-BSS-COR
      ESS = TSS-(ASS+BSS+AXBSS)
C        COMPUTE MEAN SQUARES
      AMS = ASS/2
      BMS = BSS/3
      AXBMS = AXBSS/6
      EMS = ESS/60
C        COMPUTE F RATIOS
      FA = AMS/EMS
      FB = BMS/EMS
      FAXB = AXBMS/EMS
      RETURN
      END


      SUBROUTINE WILSON(CHISQA,CHISQB,CSQAXB)
      DIMENSION NB(3,4),XOBS(100)
      COMMON YOBS(100)
      DO 6950 I = 1,72
      XOBS(I) = YOBS(I)
 6950 CONTINUE
C        ORDER OBSERVATIONS IN YOBS VECTOR
      LIM = 71
C        INITIALIZE INT TO 1 IN EVENT ALL YOBS(I) ARE IN ORDER
 7000 INT = 1
      DO 7100 I = 1,LIM
C        IF YOBS(I+1).LT.YOBS(I) INTERCHANGE THEM
      IF(YOBS(I+1).GE.YOBS(I)) GO TO 7100
      TEMP = YOBS(I+1)
      YOBS(I+1) = YOBS(I)
      YOBS(I) = TEMP
      INT = I
C        INT GIVES LOCATION OF LAST INTERCHANGE. ALL NUMBERS BEYOND
C        YOBS(INT) ARE IN ORDER.
 7100 CONTINUE
C        IF INT = 1,NO INTERCHANGES BEYOND YOBS(1) AND YOBS(2) HAVE
C        OCCURED. ALL NUMBERS ARE IN ORDER.
      IF(INT.EQ.1) GO TO 7200
```

```
      LIM = INT-1
      GO TO 7000
 7200 XMED = (YOBS(36)+YOBS(37))/2
      NBSUM = 0
      DO 7600 I = 1,6
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7600 CONTINUE
      NB(1,1) = NBSUM
      NBSUM = 0
      DO 7700 I = 7,12
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7700 CONTINUE
      NB(1,2) = NBSUM
      NBSUM = 0
      DO 7800 I = 13,18
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7800 CONTINUE
      NB(1,3) = NBSUM
      NBSUM = 0
      DO 7900 I = 19,24
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7900 CONTINUE
      NB(1,4) = NBSUM
      NBSUM = 0
      DO 8000 I = 25,30
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8000 CONTINUE
      NB(2,1) = NBSUM
      NBSUM = 0
      DO 8100 I = 31,36
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8100 CONTINUE
      NB(2,2) = NBSUM
      NBSUM = 0
      DO 8200 I = 37,42
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8200 CONTINUE
      NB(2,3) = NBSUM
      NBSUM = 0
      DO 8300 I = 43,48
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8300 CONTINUE
      NB(2,4) = NBSUM
      NBSUM = 0
      DO 8400 I = 49,54
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8400 CONTINUE
      NB(3,1) = NBSUM
```

```
      NBSUM = 0
      DO 8500 I = 55,60
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8500 CONTINUE
      NB(3,2) = NBSUM
      NBSUM = 0
      DO 8600 I = 61,66
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8600 CONTINUE
      NB(3,3) = NBSUM
      NBSUM = 0
      DO 8700 I = 67,72
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8700 CONTINUE
      NB(3,4) = NBSUM
      NBF1D = NB(1,1)+NB(1,2)+NB(1,3)+NB(1,4)
      NBF2D = NB(2,1)+NB(2,2)+NB(2,3)+NB(2,4)
      NBF3D = NB(3,1)+NB(3,2)+NB(3,3)+NB(3,4)
      NBFD1 = NB(1,1)+NB(2,1)+NB(3,1)
      NBFD2 = NB(1,2)+NB(2,2)+NB(3,2)
      NBFD3 = NB(1,3)+NB(2,3)+NB(3,3)
      NBFD4 = NB(1,4)+NB(2,4)+NB(3,4)
      NBDD = NBFD1+NBFD2+NBFD3+NBFD4
C        COMPUTATIONS FOR CHISQUARE TESTS
      TMULT = (4.0*3.0*4.0)/72.0
      NTCOR = 3
      CHISQT = TMULT*((NB(1,1)-3)**2+(NB(1,2)-3)**2+(NB(1,3)-3)**2+(NB(1
     1,4)-3)**2+(NB(2,1)-3)**2+(NB(2,2)-3)**2+(NB(2,3)-3)**2+(NB(2,4)-3)
     1**2+(NB(3,1)-3)**2+(NB(3,2)-3)**2+(NB(3,3)-3)**2+(NB(3,4)-3)**2)
      AMULT = (4.0*3.0)/72.0
      NACOR = 72/(2*3)
      CHISQA = AMULT*((NBF1D-NACOR)**2+(NBF2D-NACOR)**2+(NBF3D-NACOR)**2
     1)
      BMULT = (4.0*4.0)/72.0
      NBCOR = 72/(2*4)
      CHISQB = BMULT*((NBFD1-NBCOR)**2+(NBFD2-NBCOR)**2+(NBFD3-NBCOR)**2
     1+(NBFD4-NBCOR)**2)
      CSQAXB = CHISQT-CHISQA-CHISQB
      NADF = 2
      NBDF = 3
      NAXBDF = 6
      RETURN
      END
```

COMPUTER PROGRAM 5: SKEWED #1 HISTOGRAM

```
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
      IX = 97531
      SIGMA = 2.0
      EX = 2.0
      SUMNRM = 0.0
      SNORSQ = 0.0
      SNORCU = 0.0
      SNORFR = 0.0
      N = 266400
      DO 1800 L = 1,3700
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      SDNORM(I) = -EX*ALOG(R)
      YNORM(I) = SDNORM(I)-EX
      SUMNRM = SUMNRM+YNORM(I)
      YNORSQ(I) = YNORM(I)**2
      SNORSQ = SNORSQ + YNORSQ(I)
      YNORCU(I) = YNORSQ(I)*YNORM(I)
      YNORFR(I) = YNORSQ(I)**2
      SNORCU = SNORCU+YNORCU(I)
      SNORFR = SNORFR+YNORFR(I)
      IF(YNORM(I).GE.12.0) GO TO 41
      IF(YNORM(I).LE.-12.0) GO TO 22
      Y = ((YNORM(I)+12.0)*20.0)/24.0
      K = Y + 1
      GO TO (22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41
     1),K
   22 NBOX(1) = NBOX(1) + 1
      GO TO 50
   23 NBOX(2) = NBOX(2) + 1
      GO TO 50
   24 NBOX(3) = NBOX(3) + 1
      GO TO 50
   25 NBOX(4) = NBOX(4) + 1
      GO TO 50
   26 NBOX(5) = NBOX(5) + 1
      GO TO 50
   27 NBOX(6) = NBOX(6) + 1
      GO TO 50
   28 NBOX(7) = NBOX(7) + 1
```

```
      GO TO 50
   29 NBOX(8) = NBOX(8) + 1
      GO TO 50
   30 NBOX(9) = NBOX(9) + 1
      GO TO 50
   31 NBOX(10) = NBOX(10) + 1
      GO TO 50
   32 NBOX(11) = NBOX(11) + 1
      GO TO 50
   33 NBOX(12) = NBOX(12) + 1
      GO TO 50
   34 NBOX(13) = NBOX(13) + 1
      GO TO 50
   35 NBOX(14) = NBOX(14) + 1
      GO TO 50
   36 NBOX(15) = NBOX(15) + 1
      GO TO 50
   37 NBOX(16) = NBOX(16) + 1
      GO TO 50
   38 NBOX(17) = NBOX(17) + 1
      GO TO 50
   39 NBOX(18) = NBOX(18) + 1
      GO TO 50
   40 NBOX(19) = NBOX(19) + 1
      GO TO 50
   41 NBOX(20) = NBOX(20) + 1
   50 CONTINUE
 1800 CONTINUE
      YNMEAN = SUMNRM/N
      YNVAR = (SNORSQ - (SUMNRM**2/N))/(N - 1)
      YMOM3 = (SNORCU-(3.0*SUMNRM*SNORSQ)/N+((SUMNRM**2)*3.0*SUMNRM)/(N*
     1*2)-(SUMNRM**3)/(N**2))/(N-1)
      YMOM4 = (SNORFR-(4.0*SUMNRM*SNORCU)/N+((SUMNRM**2)*6.0*SNORSQ)/(N*
     1*2)-((SUMNRM**3)*4.0*SUMNRM)/(N**3)+(SUMNRM**4)/(N**3))/(N-1)
      BETA1 = (YMOM3**2)/(YNVAR**3)
      SBETA1 = SQRT(BETA1)
      BETA2 = YMOM4/(YNVAR**2)
      DO 3000 I = 1,20
      WRITE (6,2500)I,YNMEAN,YNVAR,NBOX(I)
 2500 FORMAT(1H,I2,10X,2F15.3,5X,I6)
 3000 CONTINUE
      WRITE(6,9500) YMOM3,YMOM4
 9500 FORMAT(1H,2F25.5)
      WRITE(6,9600) SBETA1,BETA1,BETA2
 9600 FORMAT(1H,3F15.5)
      DO 4000 J = 1,72
      WRITE(6,3500) J,YNORM(J)
 3500 FORMAT(1H,I2,10X,F15.5)
```

```
4000 CONTINUE
     STOP
     END
```

COMPUTER PROGRAM 6: SKEWED #1 POWER W/ANOV, WILSON SUB-ROUTINES

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C         INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      EX = 2.0
      N = 3700
      DO 1800 L = 1,N
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      SDNORM(I) = -EX*ALOG(R)
      YNORM(I) = SDNORM(I)-EX
   50 CONTINUE
C         MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C         MODEL CELL 2
      DO 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
```

```
C        MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CONTINUE
C        MODEL CELL 4
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C        MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CONTINUE
C        MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CONTINUE
C        MODEL FOR CELL 7
      DO 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C        MODEL FOR CELL 8
      DO 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C        MODEL FOR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C        MODEL FOR CELL 10
      DO 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C        MODEL FOR CELL 11
      DO 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CONTINUE
C        MODEL FOR CELL 12
      DO 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISQB,CSQAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION
```

```
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
 1800 CONTINUE
      XPOWA = NPASUM*1.0/(1.0*N)
      XPOWB = NPBSUM*1.0/(1.0*N)
      XPOWX = NPXSUM*1.0/(1.0*N)
      XPOWAW = MPASUM*1.0/(1.0*N)
      XPOWBW = MPBSUM*1.0/(1.0*N)
      XPOWXW = MPXSUM*1.0/(1,0*N)
      WRITE(6,4500)XPOWA,XPOWB,XPOWX
      WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
 4500 FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
      STOP
      END


      SUBROUTINE ANOV(FA,FB,FAXB)
      DIMENSION WORK(18,4),YSQ(18,4),TDJD(10),TDJDSQ(10),TIJD(20),TIJDSQ
     1(20),TIDD(10),TIDDSQ(10)
      COMMON YOBS(100)
C        CONVERT VECTOR TO 18X4 MATRIX WITH CELLS IN CORRECT ORDER.
      DO 1900 I = 1,6
      WORK(I,1) = YOBS(I)
 1900 CONTINUE
      DO 2000 I = 1,6
      K = I+6
      WORK(I,2) = YOBS(K)
 2000 CONTINUE
      DO 2100 I = 1,6
      K = I+12
      WORK(I,3) = YOBS(K)
 2100 CONTINUE
      DO 2200 I = 1,6
      K = I+18
      WORK(I,4) = YOBS(K)
 2200 CONTINUE
      DO 2300 I = 7,12
      K = I+18
      WORK(I,1) = YOBS(K)
 2300 CONTINUE
      DO 2400 I = 7,12
      K = I+24
      WORK(I,2) = YOBS(K)
 2400 CONTINUE
      DO 2500 I = 7,12
      K = I+30
      WORK(I,3) = YOBS(K)
```

```
 2500 CONTINUE
      DO 2600 I = 7,12
      K = I+36
      WORK(I,4) = YOBS(K)
 2600 CONTINUE
      DO 2700 I = 13,18
      K = I+36
      WORK(I,1) = YOBS(K)
 2700 CONTINUE
      DO 2800 I = 13,18
      K = I+42
      WORK(I,2) = YOBS(K)
 2800 CONTINUE
      DO 2900 I = 13,18
      K = I+48
      WORK(I,3) = YOBS(K)
 2900 CONTINUE
      DO 3000 I = 13,18
      K = I+54
      WORK(I,4) = YOBS(K)
 3000 CONTINUE
C         FOR LATER FINDING TSS
      DO 3100 I = 1,18
      DO 3200 J = 1,4
      YSQ(I,J) = WORK(I,J)**2
 3200 CONTINUE
 3100 CONTINUE
C         FOR LATER FINDING BSS
      DO 3300 J = 1,4
      WSUM = 0.0
      DO 3400 I = 1,18
      WSUM = WSUM+WORK(I,J)
 3400 CONTINUE
      TDJD(J) = WSUM
      TDJDSQ(J) = TDJD(J)**2
 3300 CONTINUE
C         FOR LATER FINDING ASS AND AXBSS
      ISTART = 1
      IFIN = 6
      M = 0
 3550 DO 3500 K = 1,4
      ASUM = 0.0
      M = M+1
      DO 3600 I = ISTART,IFIN
      ASUM = ASUM+WORK(I,K)
 3600 CONTINUE
      TIJD(M) = ASUM
      TIJDSQ(M) = TIJD(M)**2
```

```
3500  CONTINUE
      ISTART = ISTART+6
      IFIN = IFIN+6
      IF(ISTART.GT.13)GO TO 3700
      GO TO 3550
3700  JSTART = 1
      JFIN = 4
      DO 3800 I = 1,3
      SSUM = 0.0
      DO 3900 K = JSTART,JFIN
      SSUM = SSUM+TIJD(K)
3900  CONTINUE
      TIDD(I) = SSUM
      TIDDSQ(I) = TIDD(I)**2
      JSTART = JSTART+4
      JFIN = JFIN+4
      IF(JSTART.GT.9)GO TO 4000
3800  CONTINUE
4000  SUMYSQ = 0.0
C        COMPUTE SUMS OF SQUARES
      DO 4100 I = 1,18
      DO 4200 J = 1,4
      SUMYSQ = SUMYSQ+YSQ(I,J)
4200  CONTINUE
4100  CONTINUE
      TDDD = TDJD(1)+TDJD(2)+TDJD(3)+TDJD(4)
      TDDDSQ = TDDD**2
      COR = TDDDSQ/72
      TSS = SUMYSQ-COR
      BSS = (TDJDSQ(1)+TDJDSQ(2)+TDJDSQ(3)+TDJDSQ(4))/18 -COR
      ASS = (TIDDSQ(1)+TIDDSQ(2)+TIDDSQ(3))/24-COR
      STIJSQ = 0.0
      DO 4300 I = 1,12
      STIJSQ = STIJSQ+TIJDSQ(I)
4300  CONTINUE
      AXBSS = STIJSQ/6-ASS-BSS-COR
      ESS = TSS-(ASS+BSS+AXBSS)
C        COMPUTE MEAN SQUARES
      AMS = ASS/2
      BMS = BSS/3
      AXBMS = AXBSS/6
      EMS = ESS/60
C        COMPUTE F RATIOS
      FA = AMS/EMS
      FB = BMS/EMS
      FAXB = AXBMS/EMS
      RETURN
      END
```

```
      SUBROUTINE WILSON(CHISQA,CHISQB,CSQAXB)
      DIMENSION NB(3,4),XOBS(100)
      COMMON YOBS(100)
      DO 6950 I = 1,72
      XOBS(I) = YOBS(I)
 6950 CONTINUE
C        ORDER OBSERVATIONS IN YOBS VECTOR
      LIM = 71
C        INITIALIZE INT TO 1 IN EVENT ALL YOBS(I) ARE IN ORDER
 7000 INT = 1
      DO 7100 I = 1,LIM
C        IF YOBS(I+1).LT.YOBS(I) INTERCHANGE THEM
      IF(YOBS(I+1).GE.YOBS(I)) GO TO 7100
      TEMP = YOBS(I+1)
      YOBS(I+1) = YOBS(I)
      YOBS(I) = TEMP
      INT = I
C        INT GIVES LOCATION OF LAST INTERCHANGE. ALL NUMBERS BEYOND
C        YOBS(INT) ARE IN ORDER.
 7100 CONTINUE
C        IF INT = 1,NO INTERCHANGES BEYOND YOBS(1) AND YOBS(2) HAVE
C        OCCURED. ALL NUMBERS ARE IN ORDER.
      IF(INT.EQ.1) GO TO 7200
      LIM = INT-1
      GO TO 7000
 7200 XMED = (YOBS(36)+YOBS(37))/2
      NBSUM = 0
      DO 7600 I = 1,6
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7600 CONTINUE
      NB(1,1) = NBSUM
      NBSUM = 0
      DO 7700 I = 7,12
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7700 CONTINUE
      NB(1,2) = NBSUM
      NBSUM = 0
      DO 7800 I = 13,18
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7800 CONTINUE
      NB(1,3) = NBSUM
      NBSUM = 0
      DO 7900 I = 19,24
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 7900 CONTINUE
      NB(1,4) = NBSUM
```

```
      NBSUM = 0
      DO 8000 I = 25,30
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8000 CONTINUE
      NB(2,1) = NBSUM
      NBSUM = 0
      DO 8100 I = 31,36
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8100 CONTINUE
      NB(2,2) = NBSUM
      NBSUM = 0
      DO 8200 I = 37,42
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8200 CONTINUE
      NB(2,3) = NBSUM
      NBSUM = 0
      DO 8300 I = 43,48
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8300 CONTINUE
      NB(2,4) = NBSUM
      NBSUM = 0
      DO 8400 I = 49,54
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8400 CONTINUE
      NB(3,1) = NBSUM
      NBSUM = 0
      DO 8500 I = 55,60
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8500 CONTINUE
      NB(3,2) = NBSUM
      NBSUM = 0
      DO 8600 I = 61,66
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8600 CONTINUE
      NB(3,3) = NBSUM
      NBSUM = 0
      DO 8700 I = 67,72
      IF(XOBS(I).LT.XMED) NBSUM = NBSUM+1
 8700 CONTINUE
      NB(3,4) = NBSUM
      NBF1D = NB(1,1)+NB(1,2)+NB(1,3)+NB(1,4)
      NBF2D = NB(2,1)+NB(2,2)+NB(2,3)+NB(2,4)
      NBF3D = NB(3,1)+NB(3,2)+NB(3,3)+NB(3,4)
      NBFD1 = NB(1,1)+NB(2,1)+NB(3,1)
      NBFD2 = NB(1,2)+NB(2,2)+NB(3,2)
      NBFD3 = NB(1,3)+NB(2,3)+NB(3,3)
      NBFD4 = NB(1,4)+NB(2,4)+NB(3,4)
      NBDD = NBFD1+NBFD2+NBFD3+NBFD4
```

```
C         COMPUTATIONS FOR CHISQUARE TESTS
      TMULT = (4.0*3.0*4.0)/72.0
      NTCOR = 3
      CHISQT = TMULT*((NB(1,1)-3)**2+(NB(1,2)-3)**2+(NB(1,3)-3)**2+(NB(1
     1,4)-3)**2+(NB(2,1)-3)**2+(NB(2,2)-3)**2+(NB(2,3)-3)**2+(NB(2,4)-3)
     1**2+(NB(3,1)-3)**2+(NB(3,2)-3)**2+(NB(3,3)-3)**2+(NB(3,4)-3)**2)
      AMULT = (4.0*3.0)/72.0
      NACOR = 72/(2*3)
      CHISQA = AMULT*((NBF1D-NACOR)**2+(NBF2D-NACOR)**2+(NBF3D-NACOR)**2
     1)
      BMULT = (4.0*4.0)/72.0
      NBCOR = 72/(2*4)
      CHISQB = BMULT*((NBFD1-NBCOR)**2+(NBFD2-NBCOR)**2+(NBFD3-NBCOR)**2
     1+(NBFD4-NBCOR)**2)
      CSQAXB = CHISQT-CHISQA-CHISQB
      NADF = 2
      NBDF = 3
      NAXBDF = 6
      RETURN
      END
```

```
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
      IX = 97531
      SIGMA = 2.0
      EX = SQRT(8.0)
      A = EX/(SIGMA**2)
      SUMNRM = 0.0
      SNORSQ = 0.0
      SNORCU = 0.0
      SNORFR = 0.0
      N = 266400
      DO 1800 L = 1,3700
      DO 50 I = 1,72
      TR = 1.0
      DO 100 M = 1,2
      CALL RANDU(IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      TR = TR*RX(M)
  100 CONTINUE
      FIXIT = -ALOG(TR)
      SDNORM(I) = FIXIT/A
      YNORM(I) = SDNORM(I)-EX
  325 SUMNRM = SUMNRM+YNORM(I)
      YNORSQ(I) = YNORM(I)**2
      SNORSQ = SNORSQ + YNORSQ(I)
      YNORCU(I) = YNORSQ(I)*YNORM(I)
      YNORFR(I) = YNORSQ(I)**2
      SNORCU = SNORCU+YNORCU(I)
      SNORFR = SNORFR+YNORFR(I)
      IF(YNORM(I).GE.12.0) GO TO 41
      IF(YNORM(I).LE.-12.0) GO TO 22
      Y = ((YNORM(I)+12.0)*20.0)/24.0
      K = Y + 1
      GO TO (22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41
     1),K
   22 NBOX(1) = NBOX(1) + 1
      GO TO 50
   23 NBOX(2) = NBOX(2) + 1
      GO TO 50
   24 NBOX(3) = NBOX(3) + 1
      GO TO 50
   25 NBOX(4) = NBOX(4) + 1
```

```
      GO TO 50
   26 NBOX(5) = NBOX(5) + 1
      GO TO 50
   27 NBOX(6) = NBOX(6) + 1
      GO TO 50
   28 NBOX(7) = NBOX(7) + 1
      GO TO 50
   29 NBOX(8) = NBOX(8) + 1
      GO TO 50
   30 NBOX(9) = NBOX(9) + 1
      GO TO 50
   31 NBOX(10) = NBOX(10) + 1
      GO TO 50
   32 NBOX(11) = NBOX(11) + 1
      GO TO 50
   33 NBOX(12) = NBOX(12) + 1
      GO TO 50
   34 NBOX(13) = NBOX(13) + 1
      GO TO 50
   35 NBOX(14) = NBOX(14) + 1
      GO TO 50
   36 NBOX(15) = NBOX(15) + 1
      GO TO 50
   37 NBOX(16) = NBOX(16) + 1
      GO TO 50
   38 NBOX(17) = NBOX(17) + 1
      GO TO 50
   39 NBOX(18) = NBOX(18) + 1
      GO TO 50
   40 NBOX(19) = NBOX(19) + 1
      GO TO 50
   41 NBOX(20) = NBOX(20) + 1
   50 CONTINUE
 1800 CONTINUE
      YNMEAN = SUMNRM/N
      YNVAR = (SNORSQ - (SUMNRM**2/N))/(N - 1)
      YMOM3 = (SNORCU-(3.0*SUMNRM*SNORSQ)/N+((SUMNRM**2)*3.0*SUMNRM)/(N*
     1*2)-(SUMNRM**3)/(N**2))/(N-1)
      YMOM4 = (SNORFR-(4.0*SUMNRM*SNORCU)/N+((SUMNRM**2)*6.0*SNORSQ)/(N*
     1*2)-((SUMNRM**3)*4.0*SUMNRM)/(N**3)+(SUMNRM**4)/(N**3))/(N-1)
      BETA1 = (YMOM3**2)/(YNVAR**3)
      SBETA1 = SQRT(BETA1)
      BETA2 = YMOM4/(YNVAR**2)
      DO 3000 I = 1,20
      WRITE (6,2500)I,YNMEAN,YNVAR,NBOX(I)
 2500 FORMAT(1H,I2,10X,2F15.3,5X,I6)
 3000 CONTINUE
      WRITE(6,9500) YMOM3,YMOM4
```

```
9500  FORMAT(1H,2F25.5)
      WRITE(6,9600) SBETA1,BETA1,BETA2
9600  FORMAT(1H,3F15.5)
      DO 4000 J = 1,72
      WRITE(6,3500) J,YNORM(J)
3500  FORMAT(1H,I2,10X,F15.5)
4000  CONTINUE
      STOP
      END
```

COMPUTER PROGRAM 8: SKEWED #2 POWER W/O SUBROUTINES

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C        INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      EX = SQRT(8.0)
      A = EX/(SIGMA**2)
      N = 3700
      DO 1800 L = 1,N
      DO 50 I = 1,72
      TR = 1.0
      DO 150 M = 1,2
      CALL RANDU(IX,IY,YFL)
      IX = IY
      RX(M) = YFL
      TR = TR*RX(M)
  150 CONTINUE
      FIXIT = -ALOG(TR)
      SDNORM(I) = FIXIT/A
      YNORM(I) = SDNORM(I)-EX
   50 CONTINUE
C        MODEL CELL 1
      DO 100 K = 1,6
```

```
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C        MODEL CELL 2
      DO 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
C        MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CONTINUE
C        MODEL CELL 4
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C        MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CONTINUE
C        MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CONTINUE
C        MODEL FOR CELL 7
      DO 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C        MODEL FOR CELL 8
      DO 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C        MODEL FOR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C        MODEL FOR CELL 10
      DO 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C        MODEL FOR CELL 11
      DO 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CONTINUE
C        MODEL FOR CELL 12
      DO 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
```

```
C         COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISQB,CSQAXB)
C         COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
 1800 CONTINUE
      XPOWA = NPASUM*1.0/(1.0*N)
      XPOWB = NPBSUM*1.0/(1.0*N)
      XPOWX = NPXSUM*1.0/(1.0*N)
      XPOWAW = MPASUM*1.0/(1.0*N)
      XPOWBW = MPBSUM*1.0/(1.0*N)
      XPOWXW = MPXSUM*1.0/(1.0*N)
      WRITE(6,4500)XPOWA,XPOWB,XPOWX
      WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
 4500 FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
      STOP
      END
```

COMPUTER PROGRAM 9: LEPTOKURTIC #4 HISTOGRAM

```
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
      IX = 97531
      SIGMA = 2.0
      SUMNRM = 0.0
      SNORSQ = 0.0
      SNORCU = 0.0
      SNORFR = 0.0
      N = 266400
      YMOM2 = 4.00
      BETA1 = 0.00
      BETA2 = 3.40
      XM = (5.0*BETA2-9.0)/(2.0*(BETA2-3.0))
      AASQ = (2.0*YMOM2*BETA2)/(BETA2-3.0)
      AA = SQRT(AASQ)
      ASUM = 0.0
      PIE = 3.1415927
      YO = N*GAMMA(XM)/(AA*SQRT(PIE)*GAMMA(XM-0.5))
      XVAL(1) = -11.76
      DO 150 I = 2,100
      XVAL(I) = XVAL(I-1)+0.24
  150 CONTINUE
      DO 160 I = 1,100
      XLYORD = ALOG(YO)-XM*(ALOG(1+(XVAL(I)**2/AASQ)))
      YORD(I) = EXP(XLYORD)/(1.0*N)
      ASUM = ASUM+(YORD(I)*0.24)
      CYORD(I) = ASUM
  160 CONTINUE
      DO 75 M = 101,128
      CYORD(M) = CYORD(M-1)
      XVAL(M) = XVAL(M-1)
   75 CONTINUE
      DO 1800 L = 1,3700
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      JHI = 128
      JLO = 1
  105 J = (JHI+JLO)/2
      IF(R.EQ.CYORD(J)) GO TO 320
      IF(R.GT.CYORD(J)) GO TO 315
      JX = JHI-JLO
```

```
      IF(JX.EQ.1) GO TO 310
      JHI = J
      GO TO 105
  315 JZ = JHI-JLO
      IF(JZ.EQ.1) GO TO 305
      JLO = J
      GO TO 105
  305 J = J+1
  310 H = CYORD(J)-CYORD(J-1)
      DH = R-CYORD(J-1)
      IF(H.EQ.0.0) GO TO 320
      X = DH*0.24/H
      CCR = 0.24-X
      YNORM(I) = XVAL(J)-CCR
      GO TO 325
  320 YNORM(I) = XVAL(J)
  325 SUMNRM = SUMNRM+YNORM(I)
      YNORSQ(I) = YNORM(I)**2
      SNORSQ = SNORSQ + YNORSQ(I)
      YNORCU(I) = YNORSQ(I)*YNORM(I)
      YNORFR(I) = YNORSQ(I)**2
      SNORCU = SNORCU+YNORCU(I)
      SNORFR = SNORFR+YNORFR(I)
      IF(YNORM(I).GE.12.0) GO TO 41
      IF(YNORM(I).LE.-12.0) GO TO 22
      Y = ((YNORM(I)+12.0)*20.0)/24.0
      K = Y + 1
      GO TO (22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41
     1),K
   22 NBOX(1) = NBOX(1) + 1
      GO TO 50
   23 NBOX(2) = NBOX(2) + 1
      GO TO 50
   24 NBOX(3) = NBOX(3) + 1
      GO TO 50
   25 NBOX(4) = NBOX(4) + 1
      GO TO 50
   26 NBOX(5) = NBOX(5) + 1
      GO TO 50
   27 NBOX(6) = NBOX(6) + 1
      GO TO 50
   28 NBOX(7) = NBOX(7) + 1
      GO TO 50
   29 NBOX(8) = NBOX(8) + 1
      GO TO 50
   30 NBOX(9) = NBOX(9) + 1
      GO TO 50
   31 NBOX(10) = NBOX(10) + 1
```

```
      GO TO 50
   32 NBOX(11) = NBOX(11) + 1
      GO TO 50
   33 NBOX(12) = NBOX(12) + 1
      GO TO 50
   34 NBOX(13) = NBOX(13) + 1
      GO TO 50
   35 NBOX(14) = NBOX(14) + 1
      GO TO 50
   36 NBOX(15) = NBOX(15) + 1
      GO TO 50
   37 NBOX(16) = NBOX(16) + 1
      GO TO 50
   38 NBOX(17) = NBOX(17) + 1
      GO TO 50
   39 NBOX(18) = NBOX(18) + 1
      GO TO 50
   40 NBOX(19) = NBOX(19) + 1
      GO TO 50
   41 NBOX(20) = NBOX(20) + 1
   50 CONTINUE
 1800 CONTINUE
      YNMEAN = SUMNRM/N
      YNVAR = (SNORSQ - (SUMNRM**2/N))/(N - 1)
      YMOM3 = (SNORCU-(3.0*SUMNRM*SNORSQ)/N+((SUMNRM**2)*3.0*SUMNRM)/(N*
     1*2)-(SUMNRM**3)/(N**2))/(N-1)
      YMOM4 = (SNORFR-(4.0*SUMNRM*SNORCU)/N+((SUMNRM**2)*6.0*SNORSQ)/(N*
     1*2)-((SUMNRM**3)*4.0*SUMNRM)/(N**3)+(SUMNRM**4)/(N**3))/(N-1)
      BETA1 = (YMOM3**2)/(YNVAR**3)
      SBETA1 = SQRT(BETA1)
      BETA2 = YMOM4/(YNVAR**2)
      DO 3000 I = 1,20
      WRITE (6,2500)I,YNMEAN,YNVAR,NBOX(I)
 2500 FORMAT(1H,I2,10X,2F15.3,5X,I6)
 3000 CONTINUE
      WRITE(6,9500) YMOM3,YMOM4
 9500 FORMAT(1H,2F25.5)
      WRITE(6,9600) SBETA1,BETA1,BETA2
 9600 FORMAT(1H,3F15.5)
      DO 4000 J = 1,72
      WRITE(6,3500) J,YNORM(J)
 3500 FORMAT(1H,I2,10X,F15.5)
 4000 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM 10: LEPTOKURTIC #4 POWER W/O SUBROUTINES

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C        INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      N = 266400
      NN = 3700
      YMOM2 = 4.00
      BETA1 = 0.00
      BETA2 = 3.40
      XM = (5.0*BETA2-9.0)/(2.0*(BETA2-3.0))
      AASQ = (2.0*YMOM2*BETA2)/(BETA2-3.0)
      AA = SQRT(AASQ)
      ASUM = 0.0
      PIE = 3.1415927
      YO = N*GAMMA(XM)/(AA*SQRT(PIE)*GAMMA(XM-0.5))
      XVAL(1) = -11.76
      DO 150 I = 2,100
      XVAL(I) = XVAL(I-1)+0.24
  150 CONTINUE
      DO 160 I = 1,100
      XLYORD = ALOG(YO)-XM*(ALOG(1+(XVAL(I)**2/AASQ)))
      YORD(I) = EXP(XLYORD)/(1.0*N)
```

```
      ASUM = ASUM+(YORD(I)*0.24)
      CYORD(I) = ASUM
  160 CCNTINUE
      DO 75 M = 101,128
      CYORD(M) = CYORD(M-1)
      XVAL(M) = XVAL(M-1)
   75 CONTINUE
      DO 1800 L = 1,NN
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      JHI = 128
      JLO = 1
  105 J = (JHI+JLO)/2
      IF(R.EQ.CYORD(J)) GO TO 320
      IF(R.GT.CYORD(J)) GO TO 315
      JX = JHI-JLO
      IF(JX.EQ.1) GO TO 310
      JHI = J
      GO TO 105
  315 JZ = JHI-JLO
      IF(JZ.EQ.1) GO TO 305
      JLO = J
      GO TO 105
  305 J = J+1
  310 H = CYORD(J)-CYORD(J-1)
      DH = R-CYORD(J-1)
      IF(H.EQ.0.0) GO TO 320
      X = DH*0.24/H
      COR = 0.24-X
      YNORM(I) = XVAL(J)-CCR
      GO TO 325
  320 YNORM(I) = XVAL(J)
   50 CONTINUE
C        MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C        MODEL CELL 2
      DO 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
C        MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
  300 CONTINUE
C        MODEL CELL 4
```

```
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C         MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CONTINUE
C         MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CONTINUE
C         MODEL FOR CELL 7
      DO 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C         MODEL FOR CELL 8
      DO 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C         MODEL FOR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C         MODEL FOR CELL 10
      DO 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C         MODEL FOR CELL 11
      DO 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CONTINUE
C         MODEL FOR CELL 12
      DO 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
C         COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISQB,CSQAXB)
C         COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
 1800 CONTINUE
      XPOWA = NPASUM*1.0/(1.0*NN)
```

```
            XPOWB = NPBSUM*1.0/(1.0*NN)
            XPOWX = NPXSUM*1.0/(1.0*NN)
            XPOWAW = MPASUM*1.0/(1.0*NN)
            XPOWBW = MPBSUM*1.0/(1.0*NN)
     1      XPOWXW = MPXSUM*1.0/(1.0*NN)
            WRITE(6,4500)XPOWA,XPOWB,XPOWX
            WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
      4500  FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
            STOP
            END
```

COMPUTER PROGRAM 11: PLATYKURTIC #4 HISTOGRAM

```
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
      IX = 97531
      SIGMA = 2.0
      SUMNRM = 0.0
      SNORSQ = 0.0
      SNORCU = 0.0
      SNORFR = 0.0
      N = 266400
      YMOM2 = 4.00
      BETA1 = 0.00
      BETA2 = 2.5
      XM = (5.0*BETA2-9.0)/(2.0*(3.0-BETA2))
      AASQ = (2.0*YMOM2*BETA2)/(3.0-BETA2)
      AA = SQRT(AASQ)
      ASUM = 0.0
      PIE = 3.1415927
      YO = N*GAMMA(XM+1.5)/(AA*SQRT(PIE)*GAMMA(XM+1.0))
      XVAL(1) = -5.88
      DO 150 I = 2,100
      XVAL(I) = XVAL(I-1)+0.12
  150 CONTINUE
      DO 160 I = 1,100
      XVALSQ = XVAL(I)**2
      IF(XVALSQ.GE.AASQ)GO TO 160
      XLYORD = ALOG(YO)+XM*(ALOG(1.0-(XVAL(I)**2/AASQ)))
      YORD(I) = EXP(XLYORD)/(1.0*N)
      ASUM = ASUM+(YORD(I)*0.12)
      CYORD(I) = ASUM
  160 CONTINUE
      DO 75 M = 101,128
      CYORD(M) = CYORD(M-1)
      XVAL(M) = XVAL(M-1)
   75 CONTINUE
      DO 1800 L = 1,3700
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      JHI = 128
      JLO = 1
  105 J = (JHI+JLO)/2
      IF(R.EQ.CYORD(J)) GO TO 320
```

```
      IF(R.GT.CYORD(J)) GO TO 315
      JX = JHI-JLO
      IF(JX.EQ.1) GO TO 310
      JHI = J
      GO TO 105
  315 JZ = JHI-JLO
      IF(JZ.EQ.1) GO TO 305
      JLO = J
      GO TO 105
  305 J = J+1
  310 H = CYORD(J)-CYORD(J-1)
      DH = R-CYORD(J-1)
      IF(H.EQ.0.0) GO TO 320
      X = DH*0.12/H
      COR = 0.12-X
      YNORM(I) = XVAL(J)-COR
      GO TO 325
  320 YNORM(I) = XVAL(J)
  325 SUMNRM = SUMNRM+YNORM(I)
      YNORSQ(I) = YNORM(I)**2
      SNORSQ = SNORSQ + YNORSQ(I)
      YNORCU(I) = YNORSQ(I)*YNORM(I)
      YNORFR(I) = YNORSQ(I)**2
      SNORCU = SNORCU+YNORCU(I)
      SNORFR = SNORFR+YNORFR(I)
      IF(YNORM(I).GE.6.0) GO TO 41
      IF(YNORM(I).LE.-6.0) GO TO 22
      Y = ((YNORM(I)+6.0)*20.0)/12.0
      K = Y + 1
      GO TO (22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41
     1),K
   22 NBOX(1) = NBOX(1) + 1
      GO TO 50
   23 NBOX(2) = NBOX(2) + 1
      GO TO 50
   24 NBOX(3) = NBOX(3) + 1
      GO TO 50
   25 NBOX(4) = NBOX(4) + 1
      GO TO 50
   26 NBOX(5) = NBOX(5) + 1
      GO TO 50
   27 NBOX(6) = NBOX(6) + 1
      GO TO 50
   28 NBOX(7) = NBOX(7) + 1
      GO TO 50
   29 NBOX(8) = NBOX(8) + 1
      GO TO 50
   30 NBOX(9) = NBOX(9) + 1
```

```
      GO TO 50
   31 NBOX(10) = NBOX(10) + 1
      GO TO 50
   32 NBOX(11) = NBOX(11) + 1
      GO TO 50
   33 NBOX(12) = NBOX(12) + 1
      GO TO 50
   34 NBOX(13) = NBOX(13) + 1
      GO TO 50
   35 NBOX(14) = NBOX(14) + 1
      GO TO 50
   36 NBOX(15) = NBOX(15) + 1
      GO TO 50
   37 NBOX(16) = NBOX(16) + 1
      GO TO 50
   38 NBOX(17) = NBOX(17) + 1
      GO TO 50
   39 NBOX(18) = NBOX(18) + 1
      GO TO 50
   40 NBOX(19) = NBOX(19) + 1
      GO TO 50
   41 NBOX(20) = NBOX(20) + 1
   50 CONTINUE
 1800 CONTINUE
      YNMEAN = SUMNRM/N
      YNVAR = (SNORSQ - (SUMNRM**2/N))/(N - 1)
      YMOM3 = (SNORCU-(3.0*SUMNRM*SNORSQ)/N+((SUMNRM**2)*3.0*SUMNRM)/(N*
     1*2)-(SUMNRM**3)/(N**2))/(N-1)
      YMOM4 = (SNORFR-(4.0*SUMNRM*SNORCU)/N+((SUMNRM**2)*6.0*SNORSQ)/(N*
     1*2)-((SUMNRM**3)*4.0*SUMNRM)/(N**3)+(SUMNRM**4)/(N**3))/(N-1)
      BETA1 = (YMOM3**2)/(YNVAR**3)
      SBETA1 = SQRT(BETA1)
      BETA2 = YMOM4/(YNVAR**2)
      DO 3000 I = 1,20
      WRITE (6,2500)I,YNMEAN,YNVAR,NBOX(I)
 2500 FORMAT(1H,I2,10X,2F15.3,5X,I6)
 3000 CONTINUE
      WRITE(6,9500) YMOM3,YMOM4
 9500 FORMAT(1H,2F25.5)
      WRITE(6,9600) SBETA1,BETA1,BETA2
 9600 FORMAT(1H,3F15.5)
      DO 4000 J = 1,72
      WRITE(6,3500) J,YNORM(J)
 3500 FORMAT(1H,I2,10X,F15.5)
 4000 CONTINUE
      STOP
      END
```

COMPUTER PROGRAM 12: PLATYKURTIC #4 POWER W/O SUBROUTINES

```
      IMPLICIT INTEGER*4(Z)
      DIMENSION UNIF(100),RX(100),SDNORM(100),XVAL(130),YORD(130),CYORD(
     1130),YNORM(130),YNORSQ(130),NBOX(20),YNORCU(130),YNORFR(130)
      COMMON YOBS(100)
      DATA XVAL/130*0.0/,YORD/130*0.0/,CYORD/130*0.0/,NBOX/20*0/
C        INITIAL DATA FOR RANDOM NUMBER GENERATOR AND CELL MODELS
      A1 = -1.03
      A2 = 0.28
      B1 = -1.03
      B2 = -1.03
      B3 = 1.03
      AB11 = 1.4832
      AB12 = 0.4944
      AB13 = -0.4944
      AB21 = -0.4032
      AB22 = -0.1344
      AB23 = 0.1344
      YMEAN = 100.00
      SIGMA = 2.00
      IX = 97531
      NPASUM = 0
      NPBSUM = 0
      NPXSUM = 0
      MPASUM = 0
      MPBSUM = 0
      MPXSUM = 0
      N = 266400
      MN = 3700
      YMOM2 = 4.00
      BETA1 = 0.00
      BETA2 = 2.5
      XM = (5.0*BETA2-9.0)/(2.0*(3.0-BETA2))
      AASQ = (2.0*YMOM2*BETA2)/(3.0-BETA2)
      AA = SQRT(AASQ)
      ASUM = 0.0
      PIE = 3.1415927
      YO = N*GAMMA(XM+1.5)/(AA*SQRT(PIE)*GAMMA(XM+1.0))
      XVAL(1) = -5.88
      DO 150 I = 2,100
      XVAL(I) = XVAL(I-1)+0.12
  150 CONTINUE
      DO 160 I = 1,100
      XVALSQ = XVAL(I)**2
      IF(XVALSQ.GE.AASQ)GO TO 160
```

```
      XLYORD = ALOG(YO)+XM*(ALOG(1.0-(XVAL(I)**2/AASQ)))
      YORD(I) = EXP(XLYORD)/(1.0*N)
      ASUM = ASUM+(YORD(I)*0.12)
      CYORD(I) = ASUM
  160 CONTINUE
      DO 75 M = 101,128
      CYORD(M) = CYORD(M-1)
      XVAL(M) = XVAL(M-1)
   75 CONTINUE
      DO 180  L = 1,NN
      DO 50 I = 1,72
      CALL RANDU(IX,IY,YFL)
      IX = IY
      R = YFL
      JHI = 128
      JLO = 1
  105 J = (JHI+JLO)/2
      IF(R.EQ.CYORD(J)) GO TO 320
      IF(R.GT.CYORD(J)) GO TO 315
      JX = JHI-JLO
      IF(JX.EQ.1) GO TO 310
      JHI = J
      GO TO 105
  315 JZ = JHI-JLO
      IF(JZ.EQ.1) GO TO 305
      JLO = J
      GO TO 105
  305 J = J+1
  310 H = CYORD(J)-CYORD(J-1)
      DH = R-CYORD(J-1)
      IF(H.EQ.0.0) GO TO 320
      X = DH*0.12/H
      COR = 0.12-X
      YNORM(I) = XVAL(J)-COR
      GO TO 50
  320 YNORM(I) = XVAL(J)
   50 CONTINUE
C         MODEL CELL 1
      DO 100 K = 1,6
      YOBS(K) = YMEAN+A1+B1+AB11+YNORM(K)
  100 CONTINUE
C         MODEL CELL 2
      DO 200 K = 7,12
      YOBS(K) = YMEAN+A1+B2+AB12+YNORM(K)
  200 CONTINUE
C         MODEL CELL 3
      DO 300 K = 13,18
      YOBS(K) = YMEAN+A1+B3+AB13+YNORM(K)
```

```
  300 CONTINUE
C        MODEL CELL 4
      DO 400 K = 19,24
      YOBS(K) = YMEAN+A1-B1-B2-B3-AB11-AB12-AB13+YNORM(K)
  400 CONTINUE
C        MODEL CELL 5
      DO 500 K = 25,30
      YOBS(K) = YMEAN+A2+B1+AB21+YNORM(K)
  500 CONTINUE
C        MODEL CELL 6
      DO 600 K = 31,36
      YOBS(K) = YMEAN+A2+B2+AB22+YNORM(K)
  600 CONTINUE
C        MODEL FOR CELL 7
      DO 700 K = 37,42
      YOBS(K) = YMEAN+A2+B3+AB23+YNORM(K)
  700 CONTINUE
C        MODEL FOR CELL 8
      DO 800 K = 43,48
      YOBS(K) = YMEAN+A2-B1-B2-B3-AB21-AB22-AB23+YNORM(K)
  800 CONTINUE
C        MODEL FOR CELL 9
      DO 900 K = 49,54
      YOBS(K) = YMEAN-A1-A2+B1-AB11-AB21+YNORM(K)
  900 CONTINUE
C        MODEL FOR CELL 10
      DO 1000 K = 55,60
      YOBS(K) = YMEAN-A1-A2+B2-AB12-AB22+YNORM(K)
 1000 CONTINUE
C        MODEL FOR CELL 11
      DO 1100 K = 61,66
      YOBS(K) = YMEAN-A1-A2+B3-AB13-AB23+YNORM(K)
 1100 CONTINUE
C        MODEL FOR CELL 12
      DO 1200 K = 67,72
      YOBS(K) = YMEAN-A1-A2-B1-B2-B3+AB11+AB12+AB13+AB21+AB22+AB23+YNORM
     1(K)
 1200 CONTINUE
      CALL ANOV(FA,FB,FAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION.
      IF(FA.GE.3.15)NPASUM = NPASUM+1
      IF(FB.GE.2.76)NPBSUM = NPBSUM+1
      IF(FAXB.GE.2.25)NPXSUM = NPXSUM+1
      CALL WILSON(CHISQA,CHISQB,CSQAXB)
C        COMPARE AND TOTAL FOR POWER CALCULATION
      IF(CHISQA.GE.5.99) MPASUM = MPASUM+1
      IF(CHISQB.GE.7.81) MPBSUM = MPBSUM+1
      IF(CSQAXB.GE.12.6) MPXSUM = MPXSUM+1
```

```
1800  CONTINUE
      XPOWA  = NPASUM*1.0/(1.0*NN)
      XPOWB  = NPBSUM*1.0/(1.0*NN)
      XPOWX  = NPXSUM*1.0/(1.0*NN)
      XPOWAW  = MPASUM*1.0/(1.0*NN)
      XPOWBW  = MPBSUM*1.0/(1.0*NN)
      XPOWXW  = MPXSUM*1.0/(1.0*NN)
      WRITE(6,4500)XPOWA,XPOWB,XPOWX
      WRITE(6,4500)XPOWAW,XPOWBW,XPOWXW
4500  FORMAT(1H,F8.4,5X,F8.4,5X,F8.4)
      STOP
      END
```

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ABSTRACT

The purpose of this thesis is to determine the power relationship, through computer simulation, between the parametric ANOV and non-parametric Wilson tests under controlled conditions of error non-normality.

Data is simulated using the 12 cell factorial ANOV model with three levels of factor A, four levels of factor B, and six observations per cell. Interaction is characterized such that its effect is proportional to the effect of factor A with the constant of proportionality related to factor B. Non-normality of the error term is characterized in three distribution types: skewed, leptokurtic (peaked), and platykurtic (flat). Four degrees of the three error distribution types are utilized, each related to the Pearson family of frequency curves.

Three thousand-seven hundred sets of data are generated for each degree of error type. Power is then estimated directly for both the ANOV F tests and Wilson Chi-square tests for main effects and interaction. Comparison is then made between corresponding tests showing the effect of error non-normality on the power of each.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Analysis of Variance (ANOV)						
Wilson Test						
Power						
Simulation						
Error Distribution						
Interaction						
Non-normality						

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